INVESTIGATION OF THE SENSITIVITY OF A PATROLLING SUBMARINE'S CAPABILITY TO GAIN A DETECTION AS A FUNCTION OF INCREASING SURFACE SHIP SPEEDS

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THESIS

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ABSTRACT

Numerical integration and Monte Carlo techniques are used in the development of several models in order to determine the effect on probability of random detection of a merchant ship using speeds up to 90 knots by a 10 knot submarine patrolling a back-and-forth barrier. A definite range law for detection is assumed. Individual encounter models are developed for ship tracks which cross the midpoint of the submarine patrol line at various angles. The models are extended to include the assumption of a normal-distribution of crossing points. Computer programs of the models, written in the FORTRAN IV language, are included. The results are applied in a numerical example.

It is concluded that while increases in ship speeds do result in a substantial decrease in probability of detection by a submarine in the case of a single barrier transit, a speed advantage alone when applied to a typical transit of the North Atlantic will not appreciably decrease the overall detection probability.

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TABLE OF SYMBOLS AND ABBREVIATIONS

Symbol/Abbreviation	Meaning
f_{R,Y_p}	joint density function of R and Y_p
$f(r \mid Y_p=y)$	conditional density of R, given $Y_p = y$
f_{Y_p}	marginal density function of Y_p
r	range in miles at which the submarine is able to detect any surface ship
t	variable denoting time
u	surface ship speed
v	submarine patrol speed
w	relative speed between the ship and the submarine
x_p	distance of ship from the patrol line at the start of the problem
y_p	y-coordinate of the lower extremity of the submarine patrol line
CPA	closest point of approach (sometimes referred to as "lateral range")
$E(\cdot)$	expected (mean) value of the random variable
L	length of submarine barrier patrol in miles
P(D)	probability of detection, P (detection)
R_1	initial positive vector from ship to submarine
R_n	final position vector from ship to submarine at the instant submarine changes course (R_n for this leg becomes R_1 for next leg, etc...)
T	time required for the patrolling submarine to travel a distance L at speed v

Y_p	a random variable representing the y-coordinate of the lower extremity of the submarine patrol line at time τ
α	angle of the surface ship track, measured counter-clockwise from the x-axis
Δ	indicates incremental change
ε	is an element of
$\not\varepsilon$	is not an element of
ρ	ratio of surface ship speed to submarine speed
λ	ratio of barrier patrol length to sweep width
τ	starting time for the encounter model
$\forall$	for each, for every
	union of sets, as in $A \cup B$
[ ]	closed interval
( )	open interval
( ],[ )	half-open (closed) intervals
X is $N(\mu,\sigma^2)$	X is normally-distributed with mean value μ, and standard deviation σ
$\phi(y)$	normal density function

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I. INTRODUCTION

A. BACKGROUND

1. High-Speed Ship Development

New principles and concepts in ship hull design have made possible the development of ships capable of speeds heretofore considered impossible. In particular, the advent of surface-effect ship (air cushion and captured bubble), hydrofoil, and catamaran hull designs, coupled with advances in nuclear power, gas turbine, and water-jet propulsion systems, will have revolutionary effects on naval strategy and tactics, and world shipping patterns. That serious consideration to high-speed merchantmen is being made is evidenced by the fact that the U.S. Navy and the Maritime Administration share funding in the Joint Surface-Effect Ship Program which was established in 1966 to determine the feasibility of 4,000 to 5,000-ton surface-effect ships capable of speeds in excess of 80 knots. Bell Aerosystems, Inc., and Aerojet-General Corp. have each received $1.5 million as a start on a 3-year program to build an experimental 100-ton surface-effect ship [1]. The fact that high-speed merchant ships which can enjoy large speed advantages over submarines are indeed in the offing indicates a need for reexamination of the abilities of future submarines to detect and kill these ships. This thesis makes a start by examining the effect of ship-submarine speed ratios on detection probability

resulting from random encounters by submarines on fixed patrols.

a. Literature Search

As might be expected, the literature dealing with high-speed ships is relatively sparse; those papers that do make reference to ships of the future concern themselves with the hydrofoil small-ship types [2], or with the so-called super merchant ships which have such features as large (500,000-ton) displacements, and underwater tankers of speeds up to 40 knots [3]. Various Navy studies in the past which have attended to the question of ship speeds versus vulnerability have been limited in scope to surface ship speed ranges up to only 35 knots [4, 5]. One study in particular concluded that "...When ships are sailed independently...(and) when cost is considered there is only a marginal gain in going from 20 to 24 knots...." /1/. The cost consideration at that time (1955) involved studies of conventional hulls. An earlier study [6] showed a decrease in percentage of independent merchant ships sunk from 10-12 per cent when ship speeds were approximately equal to submerged submarine speeds (4-8 knots) to 1-2 per cent when ship speeds were double that of the U-boat submerged speeds. When hull-mounted sonars on merchant ships were considered [7], maximum optimal sustained speeds for independent vessels were in the range of 20 to 24 knots.

In summary, neither the search of available studies [8], operations research papers, nor computer

searches for documentation [9] at the Defense Documentation Center, Washington, D.C., and at the U.S. Naval Postgraduate School, Monterey, California have revealed any literature regarding high-speed ships, and in particular the effect of merchant ship speed increases beyond 35 knots on vulnerability to submarines. Indeed, all facets associated with these high-speed ships should be examined, e.g., noise radiation in water, the submarine fire control problem, effect of torpedoes, etcetera.

2. <u>Vulnerability of Convoys and Independent Ships</u>

As was clearly evidenced during World Wars I and II, convoys and independent merchants were quite susceptible to attack by submarines. Allied shipping losses from the German U-boat campaign during the first five years of World War II provide sufficient evidence of the extreme vulnerability of merchant shipping [10, 11]. Considering the improvements in both submarine and submarine weapon system capabilities since this era, the shipping losses previously experienced can be regarded as the minimum losses that may be expected in a future war [10]. Furthermore, it is a foregone conclusion that in the opening stages of a future general war escort numbers will be insufficient to provide adequate convoy protection for all merchant shipping [12, 13]. It can be assumed, then, that the initial phases of a future war will see an aggressive enemy submarine force presented with many unescorted targets which can be attacked with impunity [10].

To counter the possibility of sustaining heavy merchant losses at the outset of a future war, passive defense measures such as installation of sonar on merchant ships, routing to minimize losses, convoying, and increasing merchant ship speeds have been considered [12]. This thesis considers the passive defense measure of increased merchant ship speed.

3. Future Capabilities of Submarines

The Type XXI submarine produced by the Germans in 1944 influenced basic submarine design until the appearance of the nuclear-powered submarine in 1954 [14]. Since the introduction of the nuclear-powered submarine, great technological strides have been realized in the areas of submarine hull construction, propulsion, sonar, torpedo weapons systems, and fire control systems. These advances have contributed significantly to the threat to merchant shipping posed by submarine forces of potential enemies. The modern nuclear-powered submarine can equal or exceed the speed of its surface-ship target or pursuer, and can remain submerged for days. Together with this endurance capability, the nuclear-powered submarine is able to maintain a high sustained submerged speed in a relatively quiet environment. "...The limiting lines of approach to convoys and task forces steadily widen with increasing submarine speed until there is no sector safe from attack by a 30-knot submarine...." /2/ This quote was taken from a publication dated April 30, 1962; since this time a great

deal of research and development in the area of surface-effect ships has taken place. The Joint Surface-Effect Ship Program was initiated to consider large surface-effect ships capable of high speeds. It is conceivable that in the forseeable future surface ships with speeds far exceeding the capabilities of the nuclear-powered submarine will take their place in the surface ship community. Submarine speeds are not expected to exceed their present maximum for some time to come, thus permitting the high-speed merchant ship to enjoy a comfortable advantage in speed.

4. Safety of Independents

Heretofore convoys have been recommended as the best alternative for moving merchant vessels across the oceans when a submarine threat exists [15]. This is due primarily to the fact that for slow moving (less than 15 knots) merchants, safety lies in numbers. The measure of effectiveness for analysis of merchant shipping has been percentage lost of the total number of ships in the convoy [16]. Studies completed during World War II indicated that approximately the same number of ships were lost, per attack, from large convoys as from small convoys. Hence the conclusion was to not only continue convoying, but to increase the size of the convoys as well.

With the advent of high-speed surface-effect ships, the problem of independent sailing versus convoying should again be considered. In a study conducted by the

Antisubmarine Operations Research Group Tenth Fleet in December 1943, the advantages and disadvantages that might result from independent sailings were discussed: "....Fast merchant vessels, with the exception of those carrying high priority cargo, have been able to continue sailing independently along the coast (Atlantic) much of the time." /3/ Here is a direct reference to a merchant ship speed advantage over the submarine.

Other advantages of independent sailings are of two kinds: first, the independent merchant can operate more efficiently and hence deliver more cargo in a given time; second, the reduced requirements for escorts releases these surface, air, and submerged craft for offensive roles [11].

5. Probabilities of Encounter

The problem of random encounter between two units, sometimes referred to as the searcher and the target [17, 18, 19, 20], is examined in most of the classical operations research literature, Operations Evaluation Group reports and studies (which constitute a large part of the so-called "classical literature"), and more recently by search theorists and game theorists.

Probabilities of encounters, or detections resulting from applications of search plans, are discussed in much of the literature, ranging from Kimball and Morse [16] and Koopman [18], through the summary reports of the Operations Evaluation Group [14] and applications studies [2] to the modern search theorists, such as Pollock [19, 20].

Data concerning encounters versus ship position displacement from reported submarine contacts [21] and frequency of encounters versus submarine position predictions [22] have been gathered and analyzed during the Second World War. Applications of game theory for the selection of convoy routes which would minimize ship-submarine encounters were proposed in a study by Danskin of the Operations Evaluation Group in 1954 [15], which later appeared as a paper in Operations Research [23]. The Random Encounter with no involved search effort has been studied by several authors; Koopman [17, 18] considered the random encounter of an observer progressing at a constant velocity within a uniform distribution of targets of constant speed, as well as within a circular-normal distribution of targets. The problem of random encounter between a surface force progressing at constant velocity with another unit travelling at a fixed speed, the position and heading of which are independently and normally distributed, has been examined by Dobbie in 1945 [24].

Literature concerning the effect of speed on the random encounter is limited almost exclusively to government studies and reports. Kittel, in 1944, analyzed data concerning sinkings and sightings by submarines to give an early indication of the effect of speed on the safety of ships [6]. The results were discussed by Sternhell and Thorndike in 1946 [25], and by Winston in 1955 [4]. These results, in turn, were summarized by the Operations

Evaluation Group in 1957 [12]. More recently, the general problem of the effect of speed on the vulnerability of independent merchant vessels was examined by Neufer in 1961 [4]. Indeed, Neufer appears to have been the first to attempt to determine the explicit survival probability of a merchant vessel in transit from one port to another as a function of its speed.

Finally, the probability of encounter by a unit on a barrier patrol has been examined by at least two authors. Koopman, in his Search and Screening, examined the problem using the underlying assumption of a uniform distribution of points of intersection of the target's track with the back-and-forth barrier patrol line. /4/ In particular, Koopman showed that, with a definite range law, the probability of detection by the back-and-forth barrier patrol is given by

$$P = \begin{cases} 1 - [\lambda - (\sqrt{1/\delta^2 + 1} - 1)/2]^2/\lambda(\lambda + 1), \\ \qquad \text{for } \delta \geq 1/(2\sqrt{\lambda(\lambda + 1)}) \\ 1, \qquad \text{for } \delta < 1/(2\sqrt{\lambda(\lambda + 1)}) \end{cases},$$

where

λ = ratio of the barrier patrol line length to sweep width

δ = ratio of target speed to barrier unit patrol speed

Further, Koopman showed that, for all values of λ, the back-and-forth barrier patrol is preferred to the crossover ("figure eight") type patrol for all values of $\delta \geq 1$. /5/

Van Train [26] used a war gaming method in a limited way to determine the probability of detection of a transitor intercepting a multi-unit, single or multi-line fixed barrier, in which each submarine was constrained

1) to remain in the center of an assigned zone, and

2) to conduct intercepts using headings parallel to the barrier line. The submarine's course, and position on a "starting line" were assumed to have been normally-distributed.

B. THE PROBLEM

The basic encounter problem is that of determining the probability of detection of a surface ship by a submarine on an assigned back-and-forth barrier patrol, such as that discussed by Koopman [18]. The following assumptions were made for the models:

1. Assumptions for the Basic Random Encounter Model

a. The submarine patrols a geographically-fixed back-and-forth barrier centered on a known surface ship track or convoy route.

b. The submarine patrols at a constant speed, v.

c. A definite range law for detection is used.

d. The submarine patrol length, L, is four times the detection radius.

e. The submarine position at any time, τ, is from a uniform distribution.

f. The point of intersection of the ship's track and the patrol line (or extension of the patrol line) is from a normal distribution, with standard deviation of $1/6^{th}$ of the length of the submarine patrol line.

C. METHODS OF SOLUTION

Two methods of solution to the models were devised; the first was a numerical integration technique, i.e., simulation by systematic sampling. The submarine's initial position was successively incremented by 500 yards (0.25 nautical mile) and the minimum lateral range (CPA) between the submarine and the surface vessel was computed by numerical vector analysis. The basic model considered 960 initial submarine positions for each speed ratio considered; twelve ship-submarine speed ratios, ranging from 1 to 9 were examined. The basic models also examined nine angles of intersection of tracks, ranging from 90 down to 30 degrees. The extended model considered ten specific points of intersection and eight speed ratios.

The second method may be classed as a Monte Carlo method, at least within Brown's definition: "...any procedure which involves the use of statistical sampling techniques to approximate the solution of a ... physical problem...." /6/ The Monte Carlo method utilized virtually the same computer program subroutines as the numerical integration approach, but selected the submarine's initial positions from a

uniform pseudorandom number (URN) generator [27], and selected track intersection points using the Box-Mueller method of normal random number generation [28]. For this technique, the basic model took 1,000 samples for each of nine speed ratios per crossing angle, and for 17 crossing angles. The extended model took 10,000 samples for each of nine speed ratios. The total number of vector ("maneuvering-board") solutions, then, exceeded 500,000; the usefulness of the computer in this analysis is obvious.

1. The Random Encounter Model

Consider a submarine patrolling a barrier of length L miles at a constant speed, v. Assume that the position of the submarine on its patrol line at any time, τ, is a uniformly distributed random variable, Y_b. Without loss of generality, then, the problem can be considered to begin at time τ when a surface vessel on a course α, projected to intersect the submarine patrol line at its center, is a distance $x_p/\cos\alpha$ from the point of intersection. Define the distance x_p to be equal to the range of the submarine detection (assuming a definite range law), R, which in the initial case was considered to have been 30 miles. Refer to Figure 1 and the definitions summarized below:

$\vec{u} = u \cos\alpha\ \hat{i} + u \sin\alpha\ \hat{j}$ = ship velocity;

$\vec{v} = \pm v\ \hat{j}$ = submarine velocities;

$\vec{w}_1 = \vec{v}_1 - \vec{u}$ = relative velocity of the submarine when it is on the upleg;

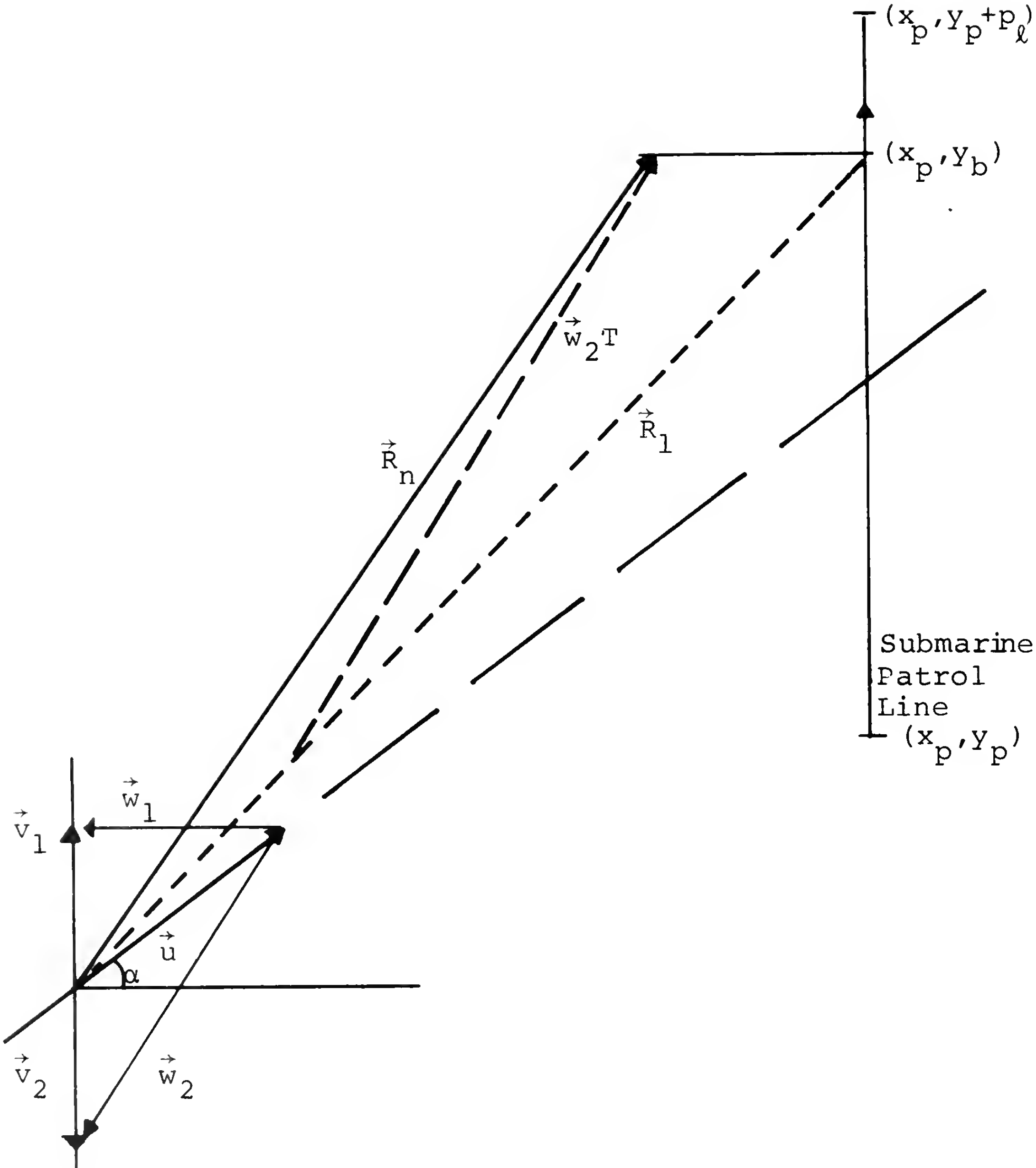


Figure 1. Vector Diagram of the Encounter (Relative Space)

$\vec{w}_2 = \vec{v}_2 - \vec{u}$ = relative velocity of the submarine when it is on the downleg;

$t_1 = (y_p + L - Y_b)/v$ = time remaining for the submarine to be on its initial leg before course reversal;

$t_1 = (Y_b - y_p)/v$ = time remaining for the submarine to be on its initial downleg before course reversal;

$T = L/v$ = length of time required for the submarine to travel entire length of its patrol line;

$\vec{R}_1 = x_p\hat{i} + Y_b\hat{j}$ = initial position vector of the submarine from the ship;

$\vec{R}_m = R_1 + \vec{w}_1 t$ = position of the submarine when it makes its course reversal.

2. <u>Algorithm to Determine Probability of Encounter</u>

(1) Select the initial value of Y_b such that $Y_b = y_p$.

(2) Specify the initial direction of submarine velocity; call it v.

(3) Compute the values of t, t_1, R_1, and R; set $\vec{w} = \vec{w}_1$.

(4) Compute the dot product $\vec{R}_1 \cdot \vec{w} = z$:

(a) If $z \geq 0$, $\overrightarrow{CPA} = \vec{R}_1$; go to step (5);

(b) If $z < 0$, compute the dot product $\vec{R}_n \cdot \vec{w} = z'$;

1) If $z' \leq 0$, $\overrightarrow{CPA} = \vec{R}_n$;

2) If $z' > 0$, the CPA lies between $\vec{R}_1$ and $\vec{R}_n$, and must be determined;

a) set $a = 1/2$;

b) compute $\vec{y} = (a\vec{w})/|w|$;

c) compute the dot product $(\vec{R}_1 + \vec{y}) \cdot \vec{w} = z''$;

1.) if z" < 0, add 1/2 to a, and go to step 2)b) above;

2.) if z" $\geq$ 0, then $\vec{CPA} = \vec{R}_1 + \vec{y}$; go to step (5).

(5) (a) If $|\vec{CPA}| < R$, a detection occurred; go to step (1) if the number of samples taken is greater than N, otherwise go to step (6).

(b) If $|\vec{CPA}| < R$, there was no detection on this leg:

1) if $\vec{w} = \vec{w}_1$, set $\vec{w} = w_2$;

2) if $\vec{w} = \vec{w}_2$, set $\vec{w} = \vec{w}_1$;

3) go to step (4).

(6) P(Encounter) = # detections/ N.

The general flow diagrams describing the logic for the models are contained in Appendix B. Note that there are two models, namely, the Basic Model, in which the ship track crosses the midpoint of the submarine patrol line at various angles, and the Extended Model, in which the crossing angle is fixed at 90 degrees, but the crossing position is varied along the track (and its extension). Appendix C contains the actual computer instructions used to generate the data analyzed herein.

D. DESCRIPTION OF THE COMPUTER SYSTEM

The computer system used in the development of this thesis was the high-performance IBM System/360 Model 67, which is located at Ingersoll Hall, U.S. Naval Postgraduate School, Monterey, California. This sophisticated data

processing system employs a FORTRAN G compiler, along with the standard software and hardware associated with the 360 system.

Programming was accomplished using the standard FORTRAN IV language as proposed by the American Standards Association X3.4.3 FORTRAN Working Group [29].

Approximate execution times for the four programs included in this thesis were as follows:

Basic Model: 45 minutes;

Extended Model: 30 minutes;

Random Sampling, Midpoint: 110 minutes;

Random Sampling, Normally-distributed crossing: 46 minutes.

II. THE RANDOM ENCOUNTER - ANALYSIS BY NUMERICAL INTEGRATION -

A. RANDOM ENCOUNTER MODEL (FIXED CROSSING POINT)

1. Description of the Model

The algorithm described in section 1.B. was used, except that the values of Y_b, and the initial direction of the submarine were determined using numerical integration techniques as follows:

The submarine's initial position for the first sample was established at the bottom of the patrol line, with the initial direction "up." The relative motion problem was solved numerically to determine if a detection occurred for this configuration. For each subsequent sample, the submarine's initial position was incremented by 500 yards (1/4th of a nautical mile), until each 500 yard increment in the initial direction had been sampled, up to the initial position lying on the patrol line extremity. The submarine direction was then reversed and the procedure continued as before except that the succeeding initial positions resulted from 500-yard increments in the new direction. The number of samples, then, was 8 x (length of the submarine patrol line in nautical miles). The basic parameters of the model from which most of the data in Appendices C and D were obtained were

submarine patrol line length: 120 miles

submarine patrol speed: 10 knots

definite range law detection range: 30 miles

These parameters were used in both the basic and extended models for the simulation by numerical integration, and for the Monte Carlo simulations.

2. Results

The data resulting from the simulation by numerical integration are contained in Appendix A; these data have been plotted in Figure 2 and Figure 3. Figure 2a shows how the detection probability varied with the ship-submarine speed ratio for a ship crossing the patrol center at various angles. The points at which detection probability was reduced by 90% and 95% have been connected by the two curves labelled "90%" and "95%," respectively.* These curves are shown in Figure 2b. Figure 3 is an expansion in scale of Figure 2b, showing greater detail. Figure 4 displays a plot of the Probability of Detection vs. Track Crossing Angle for a midpoint crossing. The behavior of P(D) as a function of α is of little practical interest for the case where a submarine commander has freedom to select a patrol line orientation with respect to a convoy route; a patrol

*The results plotted in Figure 2 seems to indicate that a barrier unit's best tactic would be to make the patrol line cross the ship's track at a shallow angle. This would indeed be the case if the ship's track were known with certainty, as was assumed in the basic model discussed here. The more realistic assumption of uncertainty in the knowledge of the ship's track makes the choice of a shallow angle undesirable from the viewpoint of the barrier unit. The design of an "optimal" barrier patrol line, the crossing angle of which depends on, say, the probability distribution of a convoy route displacement, appears to be an interesting problem, but is not considered herein.

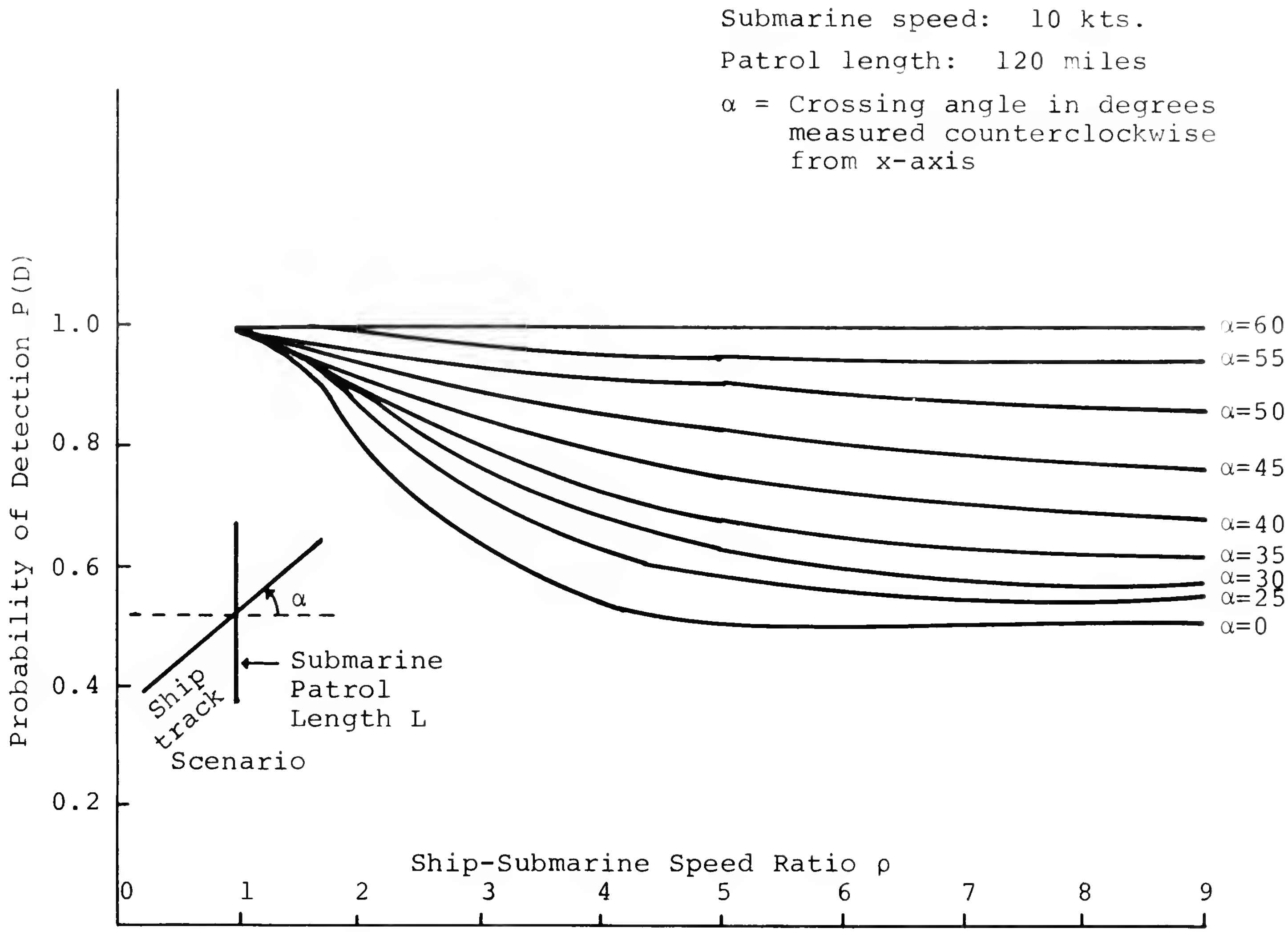


Figure 2a. Probability of Random Encounter vs. Ship-Submarine Speed Ratio (Midpoint Crossing)

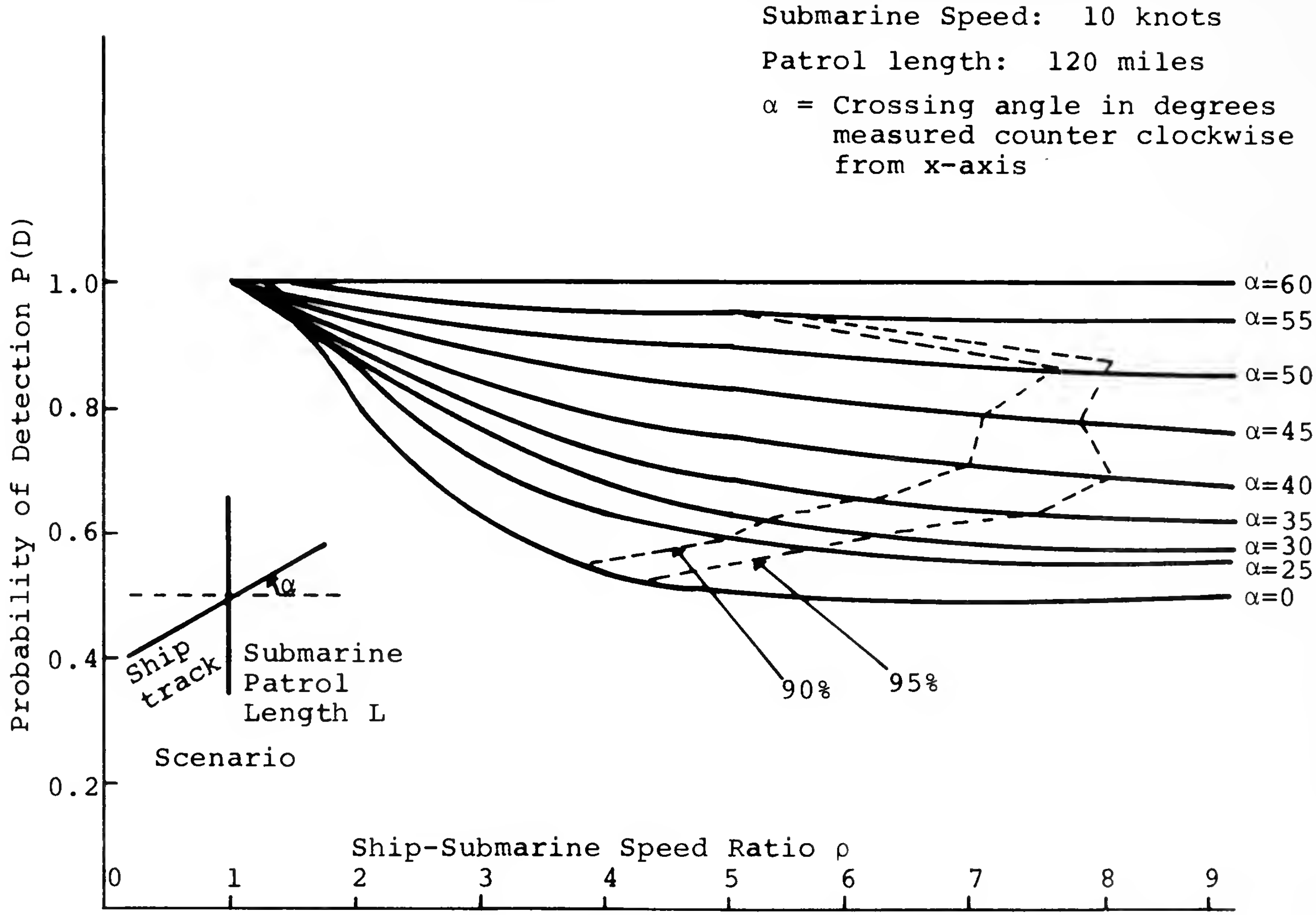


Figure 2b. Probability of Random Encounter vs. Ship-Submarine Speed Ratio (Midpoint Crossing) with 90% and 95% Detection Probability Reduction Contours

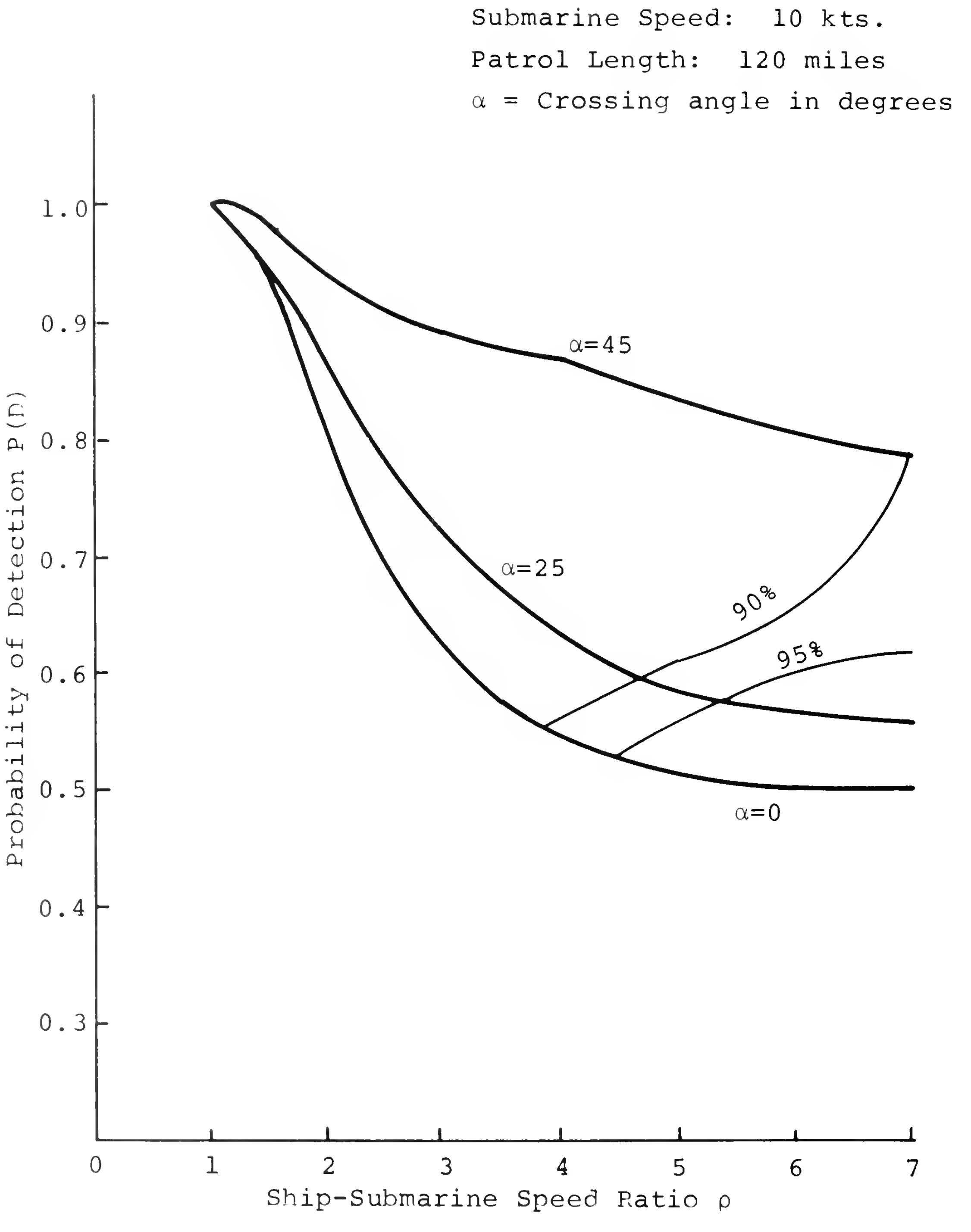


Figure 3. Probability of Random Encounter vs. Ship-Submarine Speed Ratio (Midpoint Crossing) Enlarged Scale

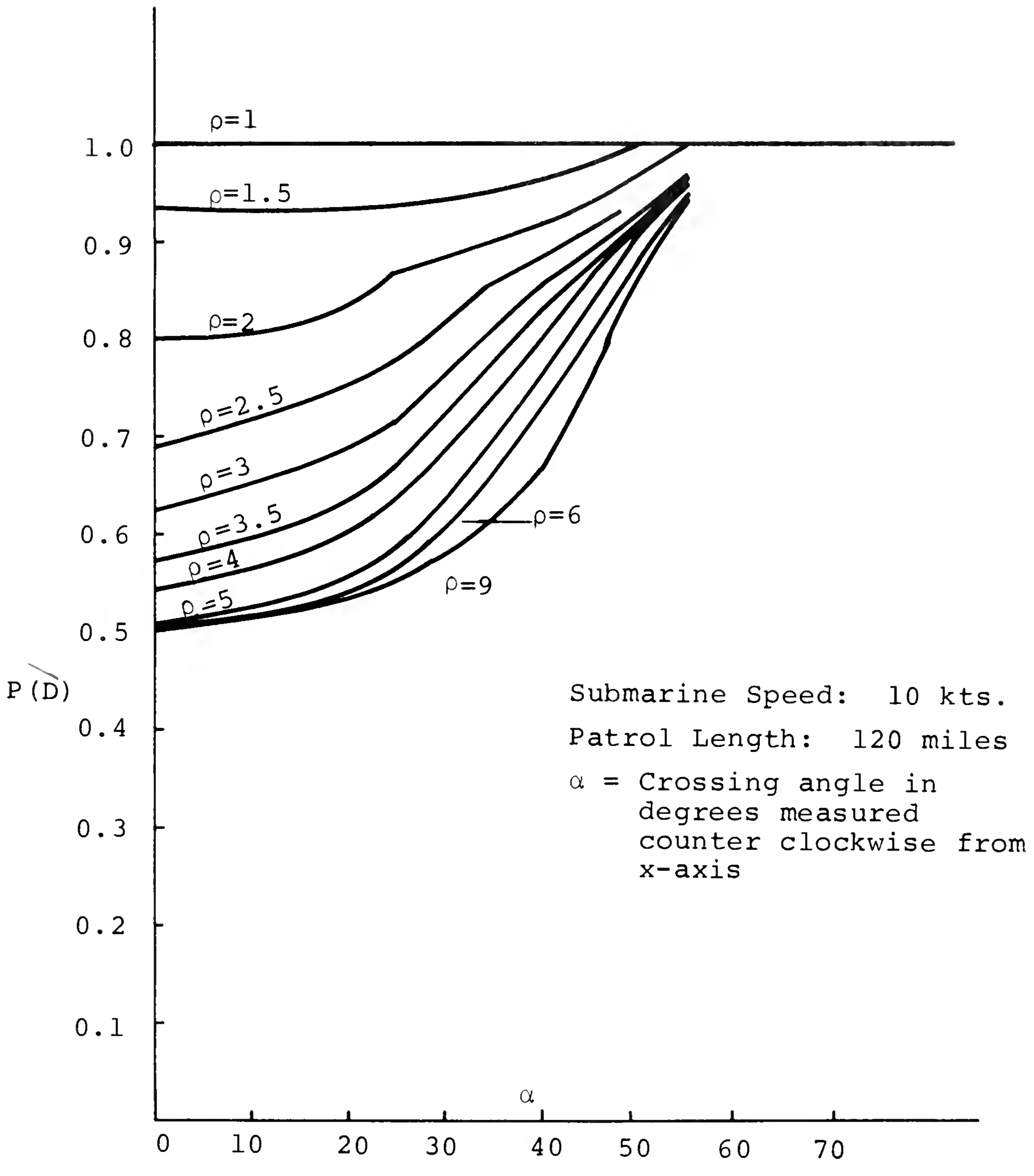


Figure 4. Probability of Random Encounter vs. Track Crossing Angle (Midpoint Crossing)

perpendicular to the expected surface ship track would normally be selected. Figure 4 could be useful if the submarine were restricted for some reason to orient its patrol line at an angle other than 90 degrees to the convoy route. The effect of relaxing the assumption that the surface ship track crosses the patrol line at the midpoint of the latter is examined in the next section. The assumption of a submarine patrol line length of 120 miles (four times the detection range) agrees with Neufer's "total attack width" which is derived in his paper. /7/

B. EXTENSION OF THE MODEL TO VARIABLE CROSSING POINTS

1. Description of the Model

The basic model contains the expression

$$y_p = x_p \tan\alpha - L/2$$

to insure that the ship's track will bisect the submarine's patrol line. It did not seem unreasonable to assume that a submarine normally attempts to center its patrol on the expected convoy route; that is, the patrol is centered on an expected value. A normal distribution of tracks, each perpendicular to the submarine patrol line, was assumed for the extension of the model, so that Y_p is a normally-distributed random variable. In terms of the coordinate system used herein,

$$E(Y_p) = -L/2$$

The logic used in the computer program is illustrated in Appendix B. The results of the computer calculations are plotted in Figure 5.

2. Analysis of Results

The flattening of the curves for the speed ratios $\rho = 1$ and $\rho = 9$ suggested the possibility of two simple limiting shapes, namely the shapes resulting when $\rho = 0$ and $\rho = \infty$. As $\rho \to 0$, i.e., when the ship's speed u is such that $u \ll v$, it can be seen that the probability of detection will approach unity. More precisely, for a given detection radius, submarine speed, and submarine patrol line length, there exists some value of ship speed such that the submarine will always have an opportunity to close within detection range. Let

L = length of the submarine patrol line

y_p = y-coordinate of the bottom limit of the submarine patrol line

u = ship speed

v = submarine speed

ρ = ship-submarine speed ratio

r = detection radius (definite range law)

$P(D)$ = probability of detection

Then, for all u, such that

$$2r/u \leq 2L/v \quad ,$$

or, for all ρ^*, such that

$$r/L \leq \rho^* \quad ,$$

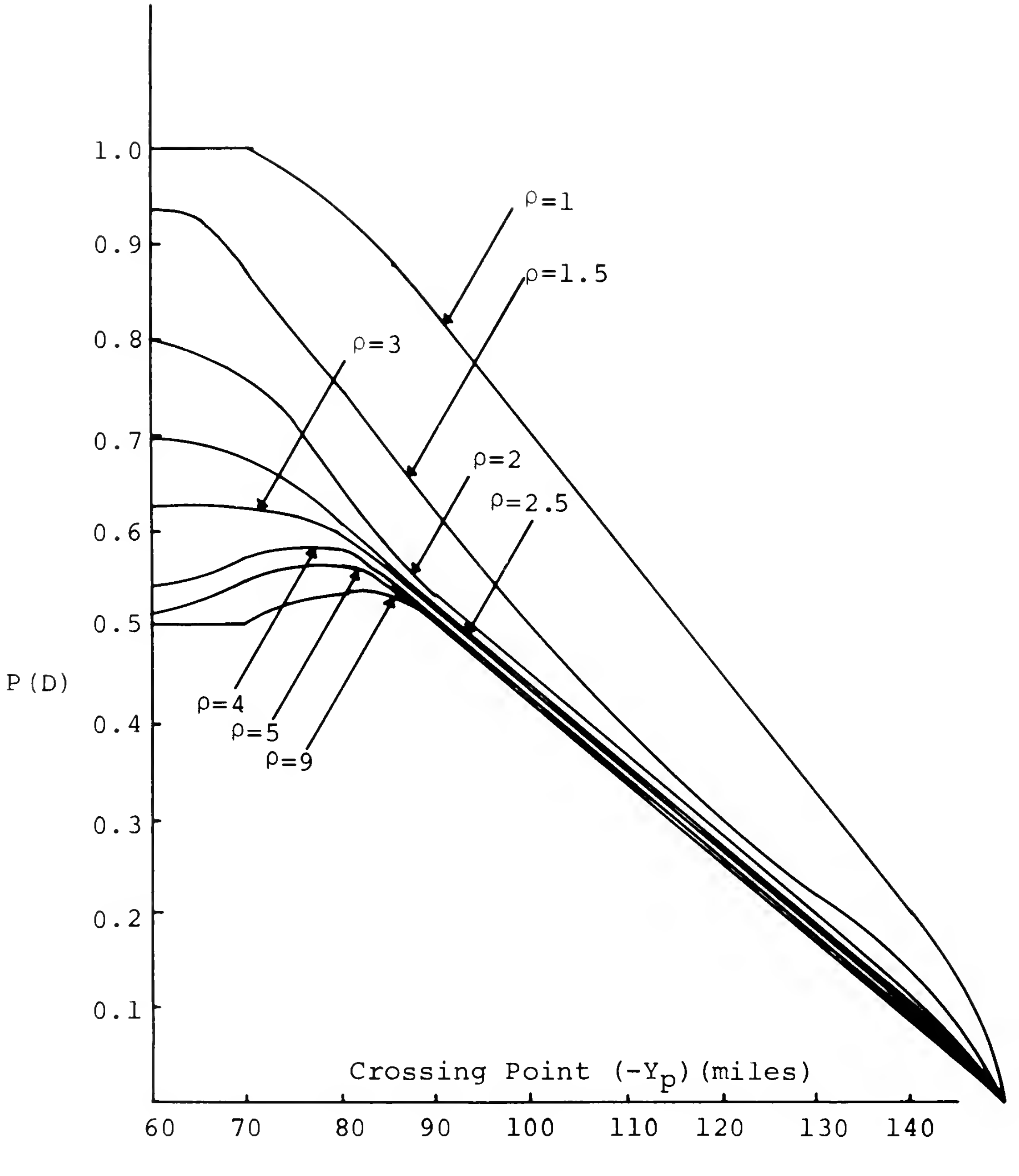


Figure 5. Probability of Detection vs. Crossing Position (Perpendicular Tracks)

$$\lim_{\rho \to \rho^*} P(D) = 1$$

if $|y_p| \leq L + r$.

For the case of $\rho \to \infty$, i.e., submarine speed approaching zero, the probability of detection is simply the probability that the submarine's position (which has been assumed to be uniformly-distributed along the patrol line) falls within distance r of the ship's track. Refer to Figure 6 below.

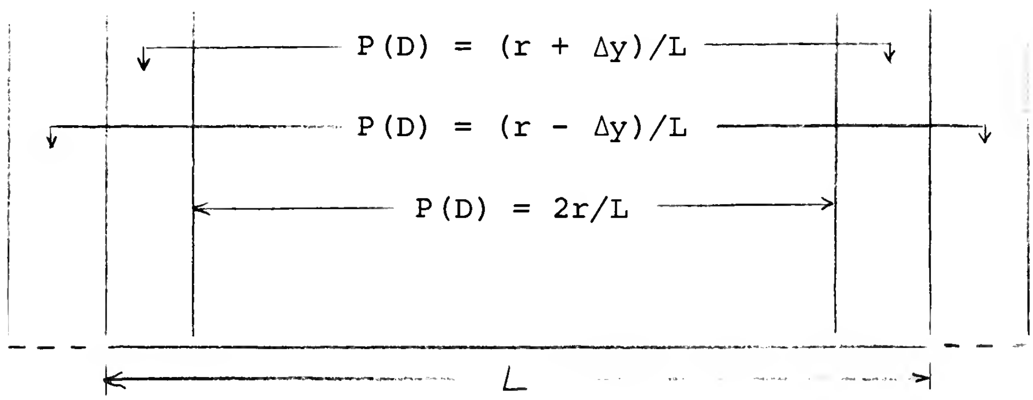


Figure 6. Probability of Detection vs. Crossing Position for Zero Submarine Speed

It can be seen from Figure 6 that if the ship's track crosses the patrol line at a distance from the patrol line end-points greater than or equal to r, then the detection probability is simply

$$2r/L$$

If the ship's track crosses the patrol line at a distance less than r, say Δy, from a patrol line end-point, then the detection probability is given by

$$(r + \Delta y)/L$$

Finally, if the ship's track does not cross the patrol line, but passes within detection range r, say Δy, of one of its end-points, the detection probability is given by

$$(r - \Delta y)/L$$

Using the convention that the coordinate system origin lies on the ship's position at the instant that the ship becomes "detectable," the above argument may be formulated as follows:

$$P(D) = \begin{cases} 2r/L, & y_p \varepsilon (-L + r, -r) \\ \dfrac{r - \min(|y_p|, |L + y_p|)\,\mathrm{sgn}[(y_p)(y_p + L)]}{L}, & \\ & y_p \varepsilon \{[-r, r] \cup [-L - r, -L + r]\} \\ 0, & y_p \varepsilon \{[r, \infty) \cup (-\infty, -L - r]\} \end{cases} \qquad (1)$$

The limiting case result, as presented herein, appears at first glance to disagree with the limiting case result obtained by Koopman; Koopman's result for a zero-speed barrier unit is P(D) = 0.333... /8/, and the result from the geometric integration for $\rho = \infty$ is 0.488. This latter value is a consequence of the assumption of normality; if Koopman's underlying assumption, namely a uniform

distribution of crossing points, were to be applied to this model, the limiting probability of detection would be (area under curve for $\rho = \infty$)x(density for uniform distribution) or,

$$\frac{(0.5)(60)}{180} + \frac{(0.5)(60)}{180} = \frac{60}{180} = 0.333\ldots \quad ,$$

which is identically Koopman's result. The model expressed by equations (1) is, therefore, consistent with Koopman. A simple FORTRAN computer program was written (Appendix F) to compute the detection probability using equations (1); the results are plotted in Figure 7. Figure 7 also contains a plot of the computed results for comparison with the limiting cases. The agreement with equations (1) is evident. The behavior of Probability of Detection vs. Crossing Position in the region of ship-submarine speed ratios greater than three is unexpected (one would expect the curves to remain uninodal). At values of ρ such that $\rho>4$, the probability of detection resulting from a patrol, the center of which was offset from the expected track crossing by about $1/6^{th}$ of the length of the patrol line, resulted in a higher detection probability than when the patrol was centered on the track crossing point. There data indicate that a submarine may effectively reduce the speed ratio by properly positioning itself.

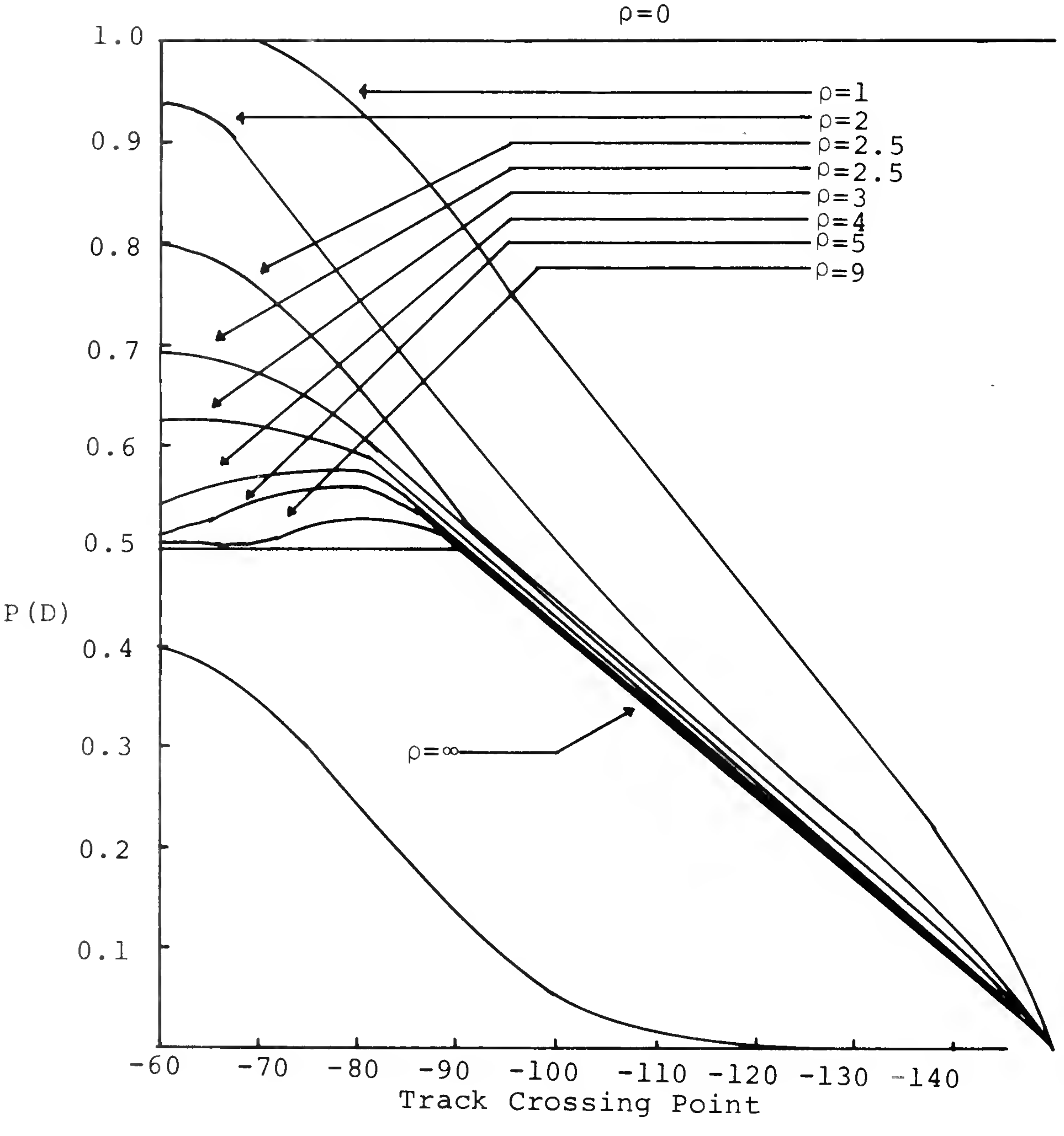


Figure 7. Probability of Detection vs. Track Crossing Point and Normal Probability Density of Track Crossing Position (Perpendicular Tracks)

TABLE I

COMPARISON OF PEAKS ON PROBABILITY CURVES
FOR SHIP-SUBMARINE SPEED RATIOS OF 5 AND 9

SPEED RATIO	P(D) FOR CENTERED PATROL LINE	P(D) FOR OFF-CENTERED PATROL LINE (20 MILES)
5	.5104	.5604
9	.5021	.5292

Table I, extracted from the data, reveals that the probability of detection of a ship with a nine-to-one speed advantage over the patrolling submarine by a submarine with a 20-mile offset patrol line is greater than the probability of detection of a ship with a five-to-one speed advantage by the same submarine with its patrol line centered on the crossing track. The table shows that with a 20-mile offset, the submarine would enjoy a higher probability of detection than it would for the case of a target with $\rho = 5$ and no offset. Figure 8 employs a contracted horizontal scale which serves to emphasize the nodal behavior discussed above. The conditional probability,

$$P(\text{Detection} \mid Y_p = y),$$

was determined by numerical analysis, and is plotted in Figure 7 for six values of ρ. The track crossing point was assumed to be normally-distributed, that is, $N(0,400)$. Now

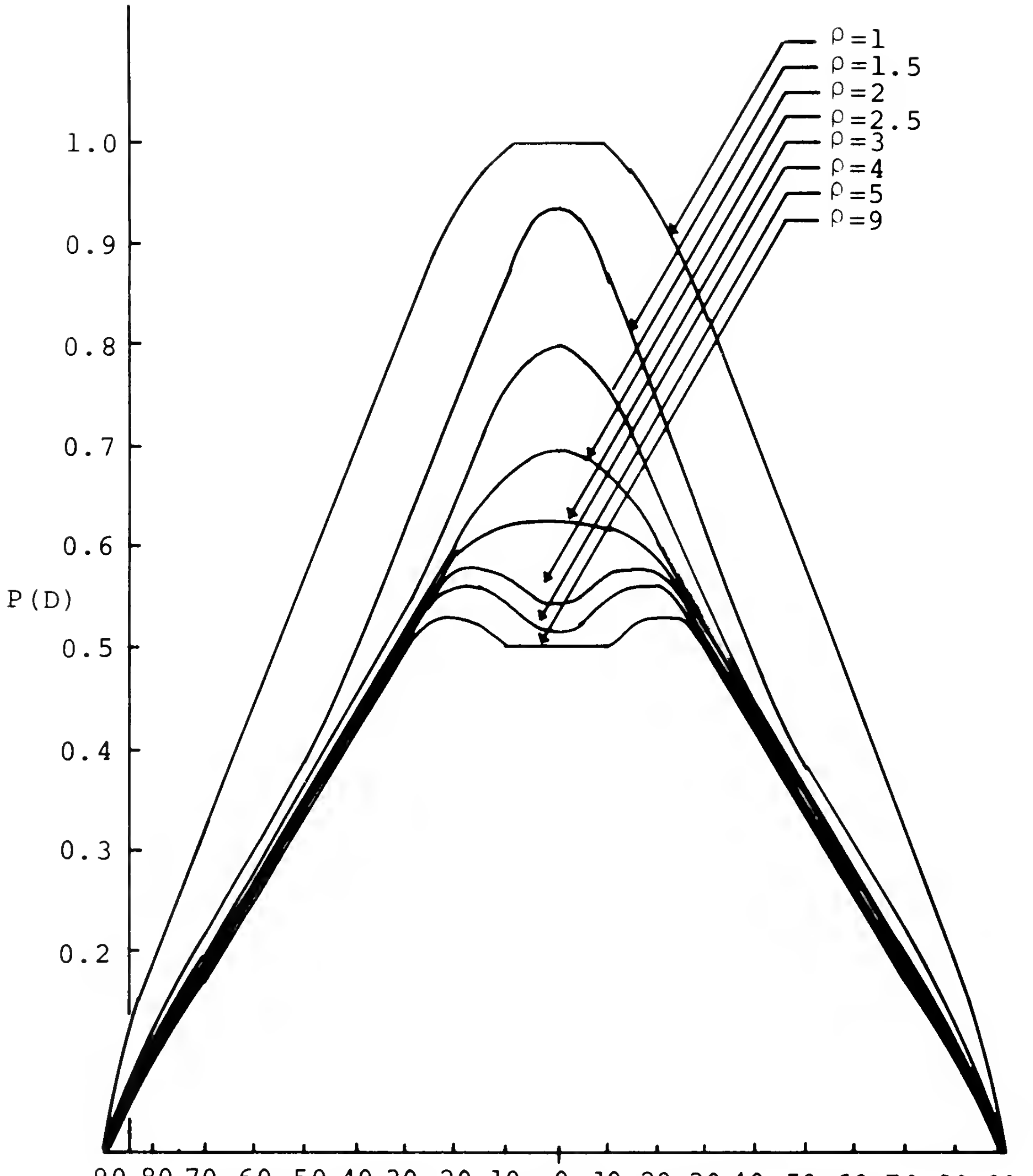


Figure 8. Probability of Detection vs. Track Crossing Position (Perpendicular Tracks)

$$P(\text{Detection}) = P(R = r),$$

where $r \varepsilon [0,\infty)$, and R is the random variable representing the distance between the submarine and the surface ship. In the problem at hand, r = 30 miles, and since a definite range law was assumed, a detection occurs when R = 30, hence

$$P(\text{Detection}) = P(D)$$

is a mass function. From Baye's formula,

$$f_{R,Y_p}(r,y) = f(r|Y_p = y) f_{Y_p}(y). \qquad (2)$$

Integrating over the range of y,

$$\int_{-\infty}^{\infty} f_{R,Y_p}(r,y)\,dy = \int_{-\infty}^{\infty} f(r|Y_p = y) f_{Y_p}(y)\,dy\ . \qquad (3)$$

The right side of equation (3) is the expectation of the conditional density $f(r|Y_p = y)$, and the left side is the marginal density, in this case the mass density, of r; hence;

$$P(D) = p_R(r) = \int_{-\infty}^{\infty} f(r\ Y_p = y) f_{Y_p}(y)\,dy\ . \qquad (4)$$

Since the conditional density in equation (4) was derived numerically, and no explicit mathematical expression was determined, the integral indicated was performed by geometric integration techniques. Appendix E shows how the area under the curve

$$z = f(r|Y_p = y) f_{Y_p}(y)$$

was obtained. Equation (4) may be rewritten as

$$\int_{-\infty}^{\infty} f(r|Y_p = y) f_{Y_p}(y)\,dy = 2\int_{0}^{L/2+r} f(r|Y_p = y) f_{Y_p}(y)\,dy, \quad (5)$$

since

$$f(r|Y_p = y) = 0, \; \forall \; y \notin [-L/2 - r, L/2 + r],$$

and the integrand is symmetric.

Figure 9 is a plot of the function to be integrated; the values of ρ used are 0, 1, 1.5, 2, 3, 5, 9, and ∞. The results of the geometrical integration are listed below in Table II, and are plotted in Figure 10.

TABLE II

GEOMETRIC INTEGRATION RESULTS

SHIP-SUBMARINE SPEED RATIO	P(D)
0	.9980
1	.9415
1.5	.7895
2	.6816
3	.5781
5	.5233
9	.5008
∞	.4885

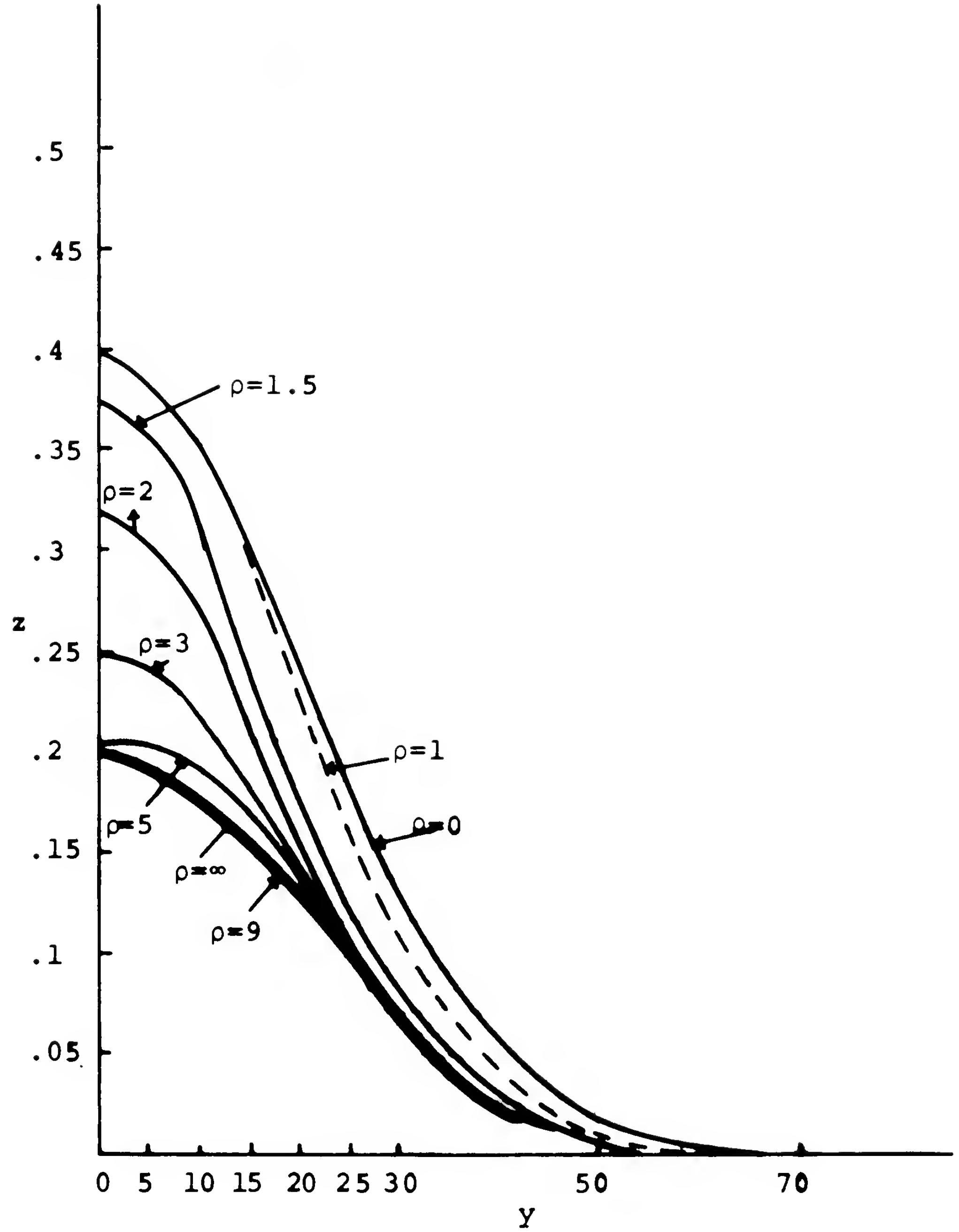


Figure 9. Plots of $z = P(D)x_p = y) \times P(Y_p = y)$ vs. y

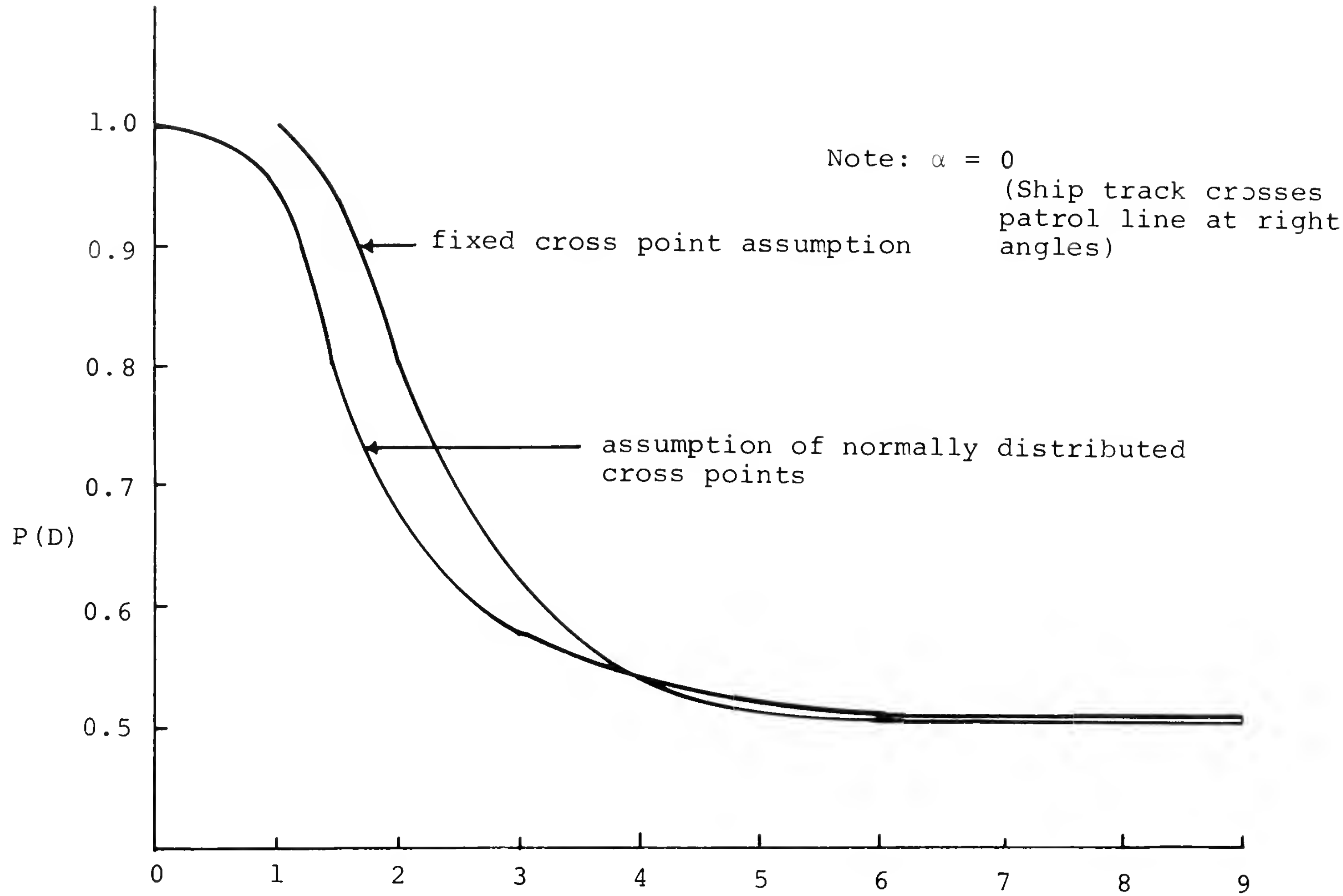


Figure 10. Comparison of Results from Basic and Extended Models P(D) vs. ρ

Cursory examination of Figure 10 reveals that for speed ratios less than four, the assumption that a fixed crossing point will yield more pessimistic results in terms of higher probabilities of detection than those arising from the assumption of normally-distributed cross points is not valid. The length of the submarine patrol line was varied from 120 miles down to 30 miles in order that the effect of changing the ratio of patrol line length to detection range might be examined. Figure 11 shows the results for a patrol line length of 90 miles (three times the detection range). Note that the binodal behavior of the probability curves which was present when a patrol line length of 120 miles was used is absent in this case, except for the situation where $\rho = 9$, where the binodal behavior is incipient. The nodal phenomenon observed indicates a possible area of interest for additional research.

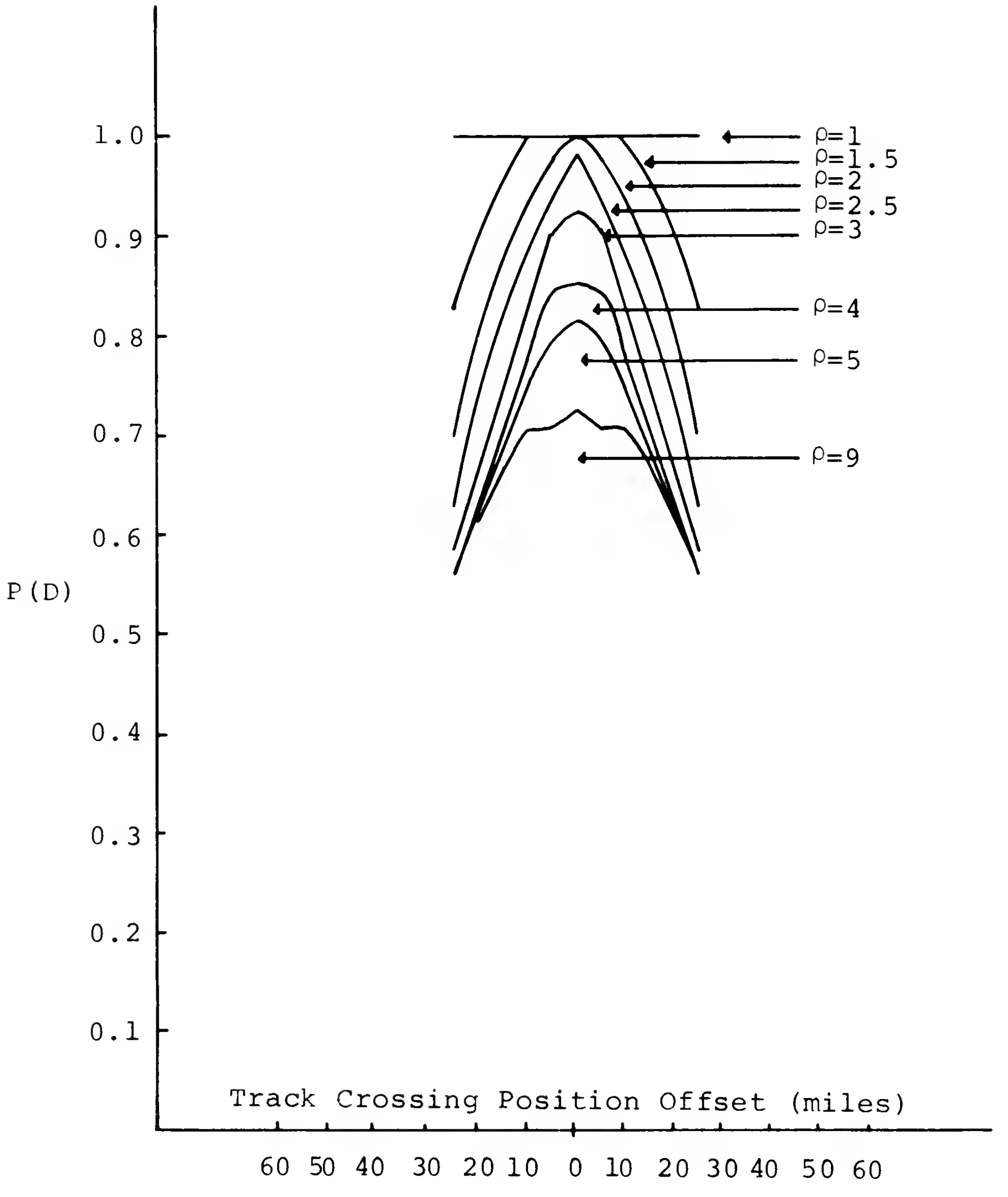


Figure 11. Probability of Detection for Various Speed Ratios vs. Track Crossing Position (Perpendicular Tracks) for L=90 miles

III. THE RANDOM ENCOUNTER - SIMULATION BY MONTE CARLO METHODS -

A. MIDPOINT CROSSING MODEL

The encounter model developed by deterministic methods was used to build the models and computer programs for the stochastic simulation. All assumptions pertaining to the deterministic model were included in the Monte Carlo approach.

The submarine's initial position on his patrol line was determined for each sample taken by selecting from a uniform (0,1) pseudorandom number generator. This procedure agreed with the assumption that the submarine was equally likely to be anywhere on his patrol line at any time τ.

The submarine's initial up or down heading on its patrol line was determined by selecting a pseudorandom number from a uniform (0,1) variate generator. If this number was less than 0.5 the submarine was assigned an initial up heading; if the number was greater than 0.5 the submarine was assigned an initial down heading. If the random variate was equal to 0.5 another uniform random variable was selected and the above procedure was repeated.

The midpoint crossing of the patrol line by the merchant was arranged by positioning the bottom of the patrol line in such a manner as to insure that this phenomenon occurred. Section (11) gives the analytical explanation dealing with the development of the deterministic approach.

The results obtained using the Monte Carlo technique are presented in Appendix A. Graphs depicting the results can be found in Figure 12. It should be noted that the results obtained utilizing the stochastic simulation agree with those obtained from the deterministic approach.

B. EXTENSION TO NORMALLY-DISTRIBUTED CROSSING POINTS

The model and computer program for normally-distributed crossing points of the convoy route about the center of the submarine patrol line is an extension of the midpoint crossing model.

The submarine's initial position and heading were determined as for the midpoint crossing model. The point of intersection of the submarine patrol line and the convoy route was determined by generating a normal N(0,400) pseudorandom variate using the Box-Mueller Method [28]. The mean of the normal variate corresponded to the center of the patrol line. The problem was then generated in a manner similar to that of the midpoint crossing model.

A relatively small sample size of 10 normal variates, one for each 1,000 uniformly-distributed initial starting positions for the submarine, was generated due to the time limitations on the computer system. This sample size took approximately 46 minutes to execute. The computer program which appears in Appendix D has been written using a larger sample size in an effort to aid in any future duplications of the simulation.

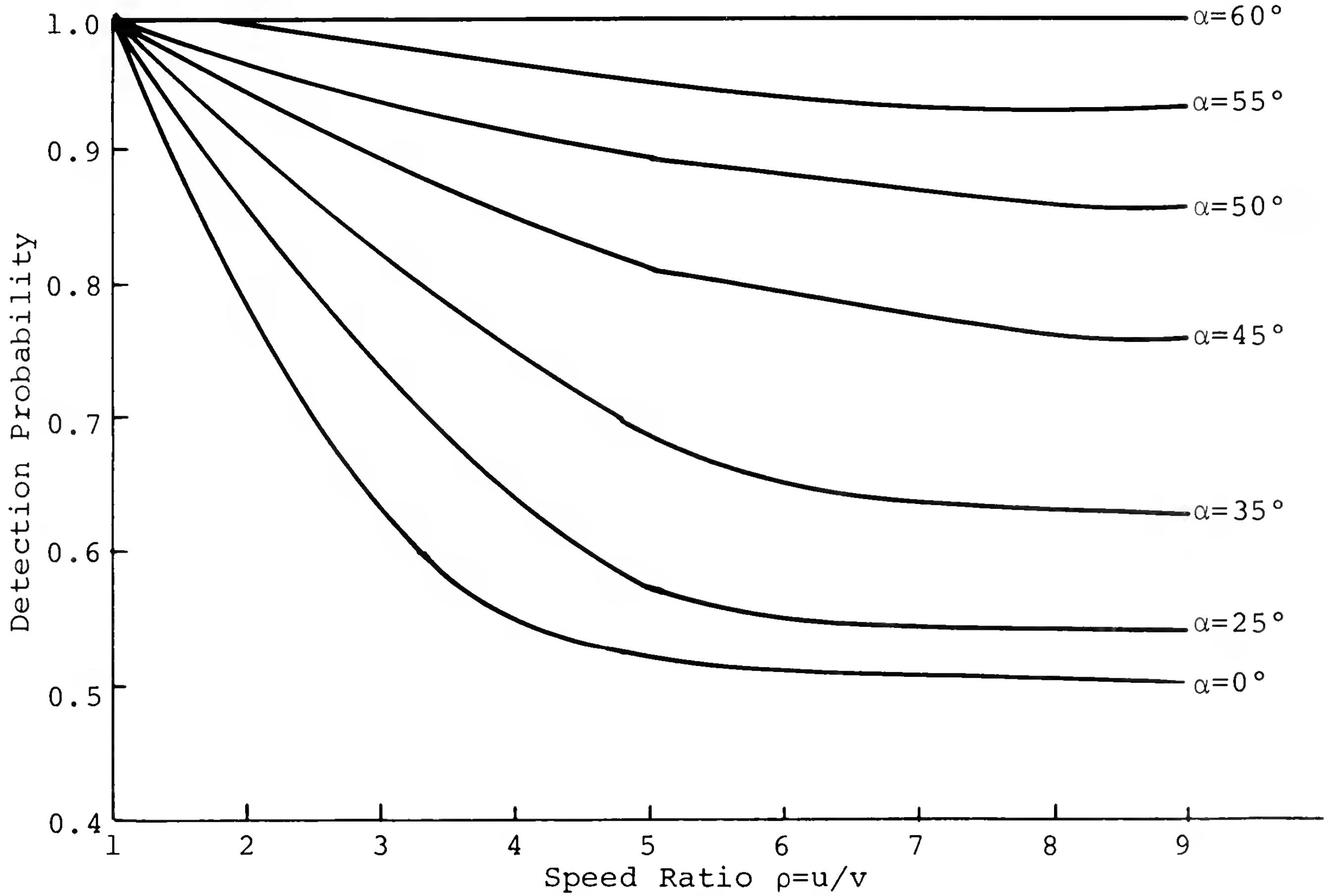


Figure 12. Detection Probability vs. Ship-Sub Speed Ratio with Parameter of Route Angle (Monte Carlo technique)

The results obtained using this simulation technique are tabulated in Appendix A. A plot of the results appears in Figure 13 and Figure 14. The results and analysis of this situation are similar to those obtained from the method of analysis by numerical integration in the extension to variable crossing points as is evident from Figure 14.

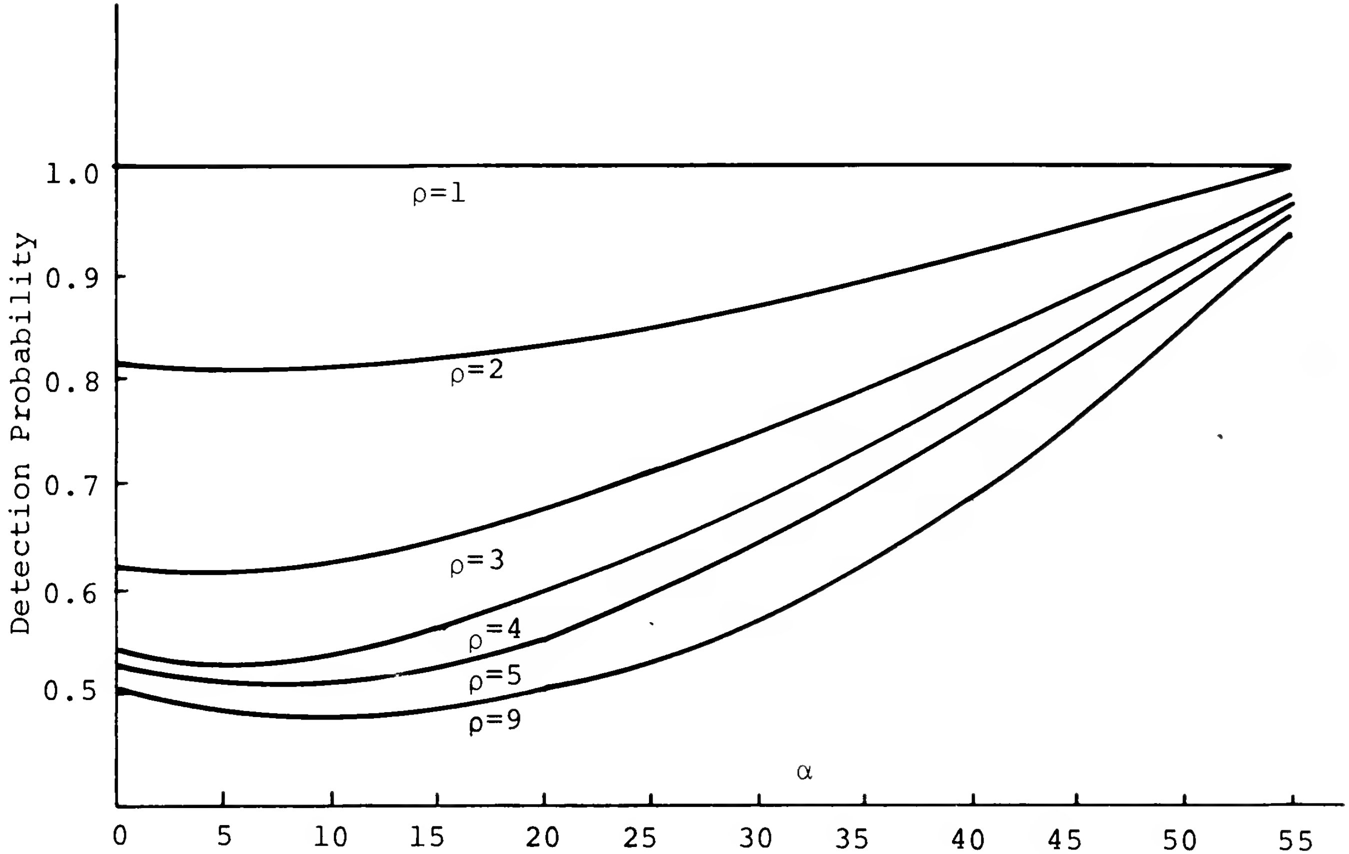


Figure 13. Detection Probability vs. Route Angle with Ship-Sub Speed Ratio as Parameter (Monte Carlo Technique)

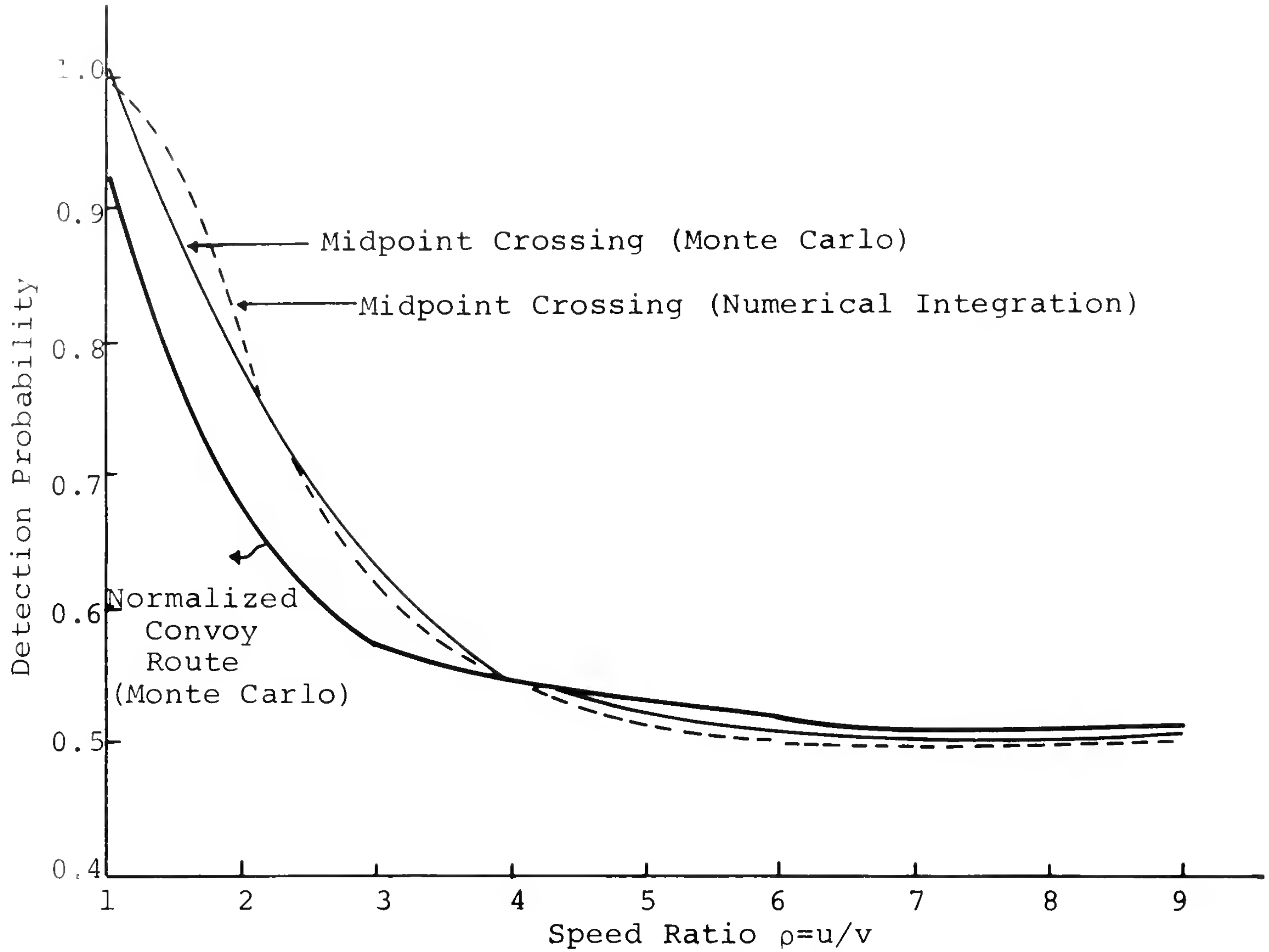


Figure 14. Detection Probability vs. Ship-Sub Speed Ratio for Route Angle $\alpha=0°$ for Midpoint Crossing and Normalized Convoy Route Approaches

IV. AN EXAMPLE OF A NORTH ATLANTIC CROSSING

A. ASSUMPTIONS

It was assumed that the major western North Atlantic seaports are New York, Norfolk, Windward Passage, and Puerto Rico (Panama Canal traffic); the major eastern North Atlantic seaports were assumed to be the Faero Islands (representing the straits between Iceland and Scotland), the English Channel, and Gibraltar. This assumption results in twelve possible Atlantic-crossing convoy routes. These possible convoy routes were used as a basis to determine the following assumptions:

1) allocation of enemy submarines
2) lengths of submarine barrier patrol lines
3) angle of orientation of the submarine patrol lines to the convoy tracks.

A consequence resulting from the assumption of specific convoy routes is a deviation from the assumptions made by Koopman [18], Neufer [5], Dobbie [24], and others which was that of a uniform distribution of submarines throughout an area. Figure 15 is a sketch which illustrates the rationale for the assumptions made herein.

Given that the only possible convoy routes are as shown in the figure, an enemy might be expected to allocate his anti-shipping submarine patrols as follows:

1. One submarine to patrol each port (since ASW activity would be expected to be heaviest near these ports,

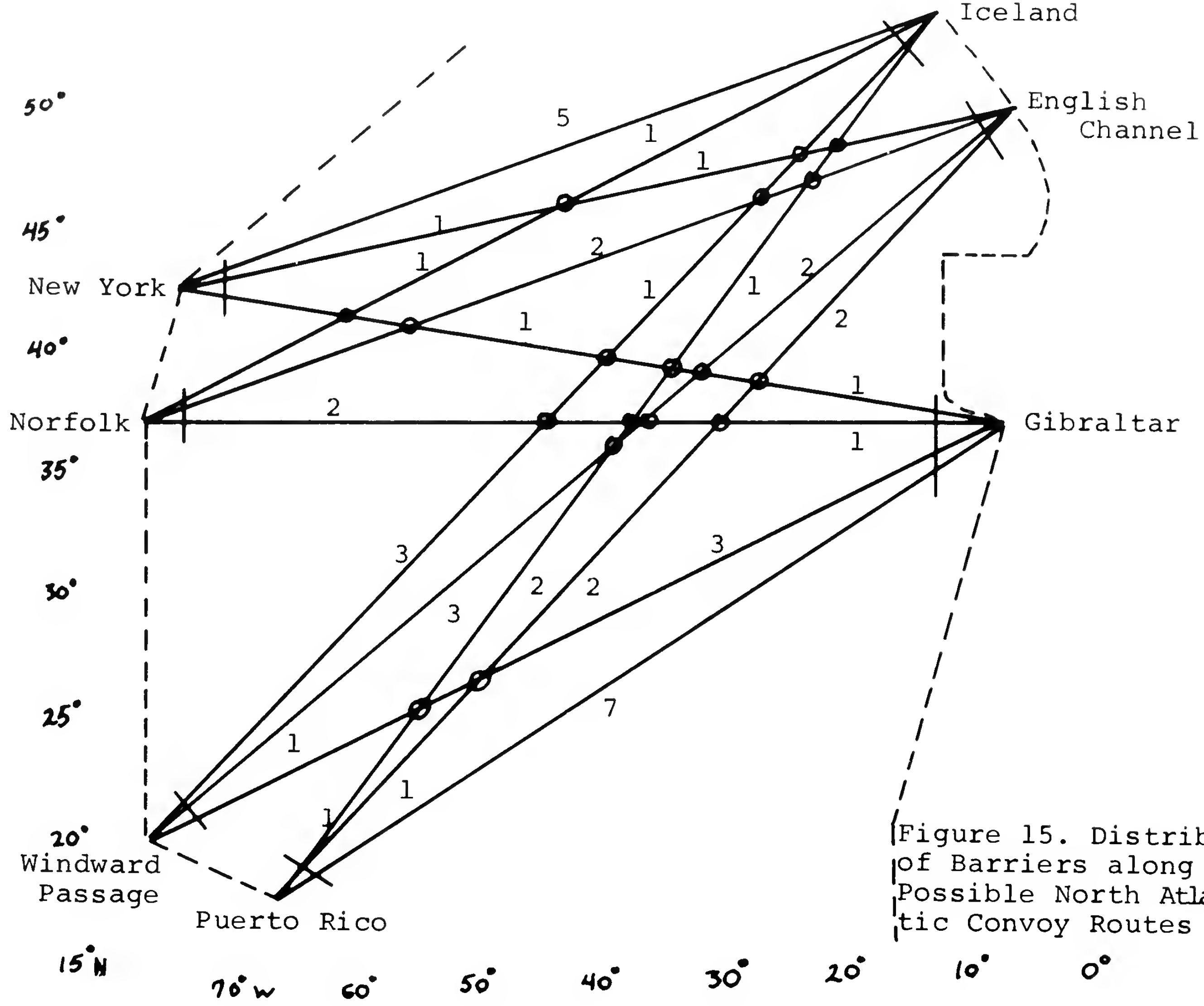


Figure 15. Distribution of Barriers along Possible North Atlantic Convoy Routes

greater numbers of patrols at these locations was not assumed).

2. A submarine patrol at each expected juncture of convoy routes (this effectively places a barrier on each of the convoy routes which merge at the junction).

3. No segment of convoy track greater than 500 miles to be unopposed by a barrier.

The preceding assumptions require that an enemy have the capability to maintain 70 submarines on station in the North Atlantic. Note that this distribution results in 105 barriers against the possible convoy routes assumed, even though only 70 submarines are employed.

Another assumption made was that the submarine barriers would be oriented according to the following rules:

1. Patrolling of straits to be oriented so as to minimize the patrol line length.

2. Patrolling of seaport exits (entrances) to be perpendicular to, and centered on, a bisector of the widest angle formed by two departure courses.

3. Patrolling of route junctions to be along the bisector of the largest angle formed by the intersection of the two convoy routes, and to be centered on the expected junction.

4. All other patrols to be centered on, and oriented at right angles to the expected route.

Additionally, all submarine patrolling speeds were assumed to be 10 knots, and all patrol lines were assumed to be

120 miles in length (unless shortened by geographical limitations).

B. APPLICATION OF THE MODELS

1. Orientation of Barriers on the Norfolk-English Channel Route

The route between Norfolk and the English Channel is approximately 3,500 miles long, and intersects the routes from New York to Gibraltar, Windward Passage to the Faero Islands, and Puerto Rico to the Faero Islands. The rules assumed required that the barrier patrols on this convoy route be given the following parameters:

TABLE III

PARAMETERS OF THE BARRIERS FOR THE EXAMPLE

BARRIER #	BARRIER LOCATION	$\alpha°$	L
1	Norfolk	5	120 miles
2	N.Y.-Gibraltar junction	12	120
3,4	Along the route	0	120
5	W.P.-Faero junction	10	120
6	P.R.-Faero junction	15	120
7	English Channel (Land's End-Ushant Is.)	0	90

2. Solution to the Example Problem

In general,

$$P(\text{Detection}) = 1 - P(\text{No Detection}),$$

and in this case,

$$P(\text{No Detection}) = P(\text{No Detection by Barrier \#1}) \times P(\text{No Detection by Barrier \#2}) \times \ldots \times P(\text{No Detection by Barrier } n) ,$$

assuming stochastic independence for simplicity. In this example, n = 7. Since the probability of detection is a function of the angle of track α, the patrol line length L, the detection range r, and the ship-submarine speed ratio ρ, the probability of detection for barrier i may be represented as

$$P_i(D) = P(D;\alpha_i,L_i,r_i,\rho_i) \quad ,$$

and

$$P(D) = \prod_{i=1}^{7} [1 - P(D;\alpha_i,L_i,r_i,\rho_i)] \quad .$$

According to the assumptions made,

$$r_1 = r_2 = \ldots = r_7 = r$$

$$\rho_1 = \rho_2 = \ldots = \rho_7 = \rho \quad ,$$

and,

$$P(\text{Detection}) = \prod_{i=1}^{7} [1 - P(D;\alpha_i,L_i,r,\rho)] \qquad (6)$$

The probabilities of detection for each of the barriers listed in Table 3 are tabulated in Table IV.

TABLE IV

P(DETECTION) FOR BARRIERS OF THE EXAMPLE

BARRIER #	P(D) VS. SHIP-SUBMARINE SPEED RATIO									
	$\rho=0$	$\rho=1$	$\rho=1.5$	$\rho=2$	$\rho=2.5$	$\rho=3$	$\rho=4$	$\rho=5$	$\rho=9$	$\rho=\infty$
1-6	.9980	.9415	.7895	.6816	.6170	.5781	.5420	.5233	.5008	.4885
7	1.	1.	1.	1.	.9861	.9250	.8528	.8139	.7250	.5000

Using the results from Table IV in equation (6),

$$1 - P(D|\rho=\rho^*) = [1 - P(D|\rho=\rho^*, L = 120, \alpha = 0, r = 30)]^6$$

$$\times[1 - P(D|\rho=\rho^*, L = 90, \alpha=0, r = 30)] \quad .$$

The results are tabulated in Table 5.

TABLE V

P(DETECTION) VERSUS SPEED FOR THE ROUTE OF THE EXAMPLE

ρ *	$P(D\|\rho=\rho^*)$
0	1.0000
1	1.0000
1.5	1.0000
2	1.0000
2.5	1.0000
3	.9996
4	.9986
5	.9978
9	.9957
∞	.9910

Table V indicates, ceterus paribus, that speed advantage alone for the surface ship will not lessen the probability of detection significantly by patrolling submarines for a typical North Atlantic crossing.

V. SUMMARY AND CONCLUSIONS

A. MONTE CARLO VERSUS NUMERICAL INTEGRATION METHODS

Figure 14 shows that close agreement exists between the Monte Carlo and numerical integration techniques; the numerical integration method is more economical in terms of computer usage time.

B. THE INDIVIDUAL RANDOM ENCOUNTER

1. Midpoint Crossing at Various Angles

Detection probability decreased from unity at $\rho \leq 1$ for all angles of intersection between the ship's track and the submarine barrier patrol line, to 0.5 at $\rho \geq 9$ (for intersection at right angles). Ninety-five per cent of the decrease occurred at speed ratios of eight and below for all angles, and for speed ratios of 4.5 and below for intersections at right angles. Stated another way, for assumed submarine speed of 10 knots, and using detection probability as the sole criterion for selecting ship speed capabilities, merchant ship speeds greater than 40 knots do not seem warranted.

2. Normally-Distributed Crossing Points

a. The assumption of normally-distributed crossing points resulted in lower detection probabilities for ship-submarine speed ratios less than four, than did the assumption of a fixed crossing point at the center of the patrol line. For speed ratios greater than four, both assumptions yielded essentially identical results.

b. If ship-submarine speed ratios equal to or greater than four are to be considered, then the binodal behavior of the probabilities of detection, illustrated in Figure 8, should be examined further. It appears that this behavior may also be a function of the ratio of submarine barrier patrol length to assumed detection range.

C. NORTH ATLANTIC TRANSIT EXAMPLE

When six or more submarine barriers are crossed by the merchant ship's track, the reduction in P(Detection) resulting from increasing the merchant ship's speed is negligible. It is evident that, from the viewpoint of decreasing random detection probability, a large speed advantage is not the answer. If a large speed advantage by a merchant ship over a patrolling submarine increases the survivability of that merchant ship, then the increase in survivability is due, not to decreased detections, but to other factors, e.g., reduced submarine weapon system effectiveness or reduced exposure time.

APPENDIX A

PROBABILITY OF RANDOM ENCOUNTER
BASIC MODEL

DEFINITE RANGE LAW
DETECTION RANGE: 30.0 MILES

LENGTH OF SUB PATROL LINE: 120.0 MILES

X-DISTANCE TO PATROL LINE: 30.0 MILES

SUBMARINE SPEED: 10.0 KNOTS

NO. OF SAMPLES TAKEN FOR EACH SCENARIO: 960

AVG.NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEEDS UPLEG	DOWNLEG	ROUTE ANGLE	PROBABILITY OF DETECTION
1.40	10.0	1.0	14.1	14.1	0	1.0000
1.36	15.0	1.5	18.0	18.0	0	0.9354
1.32	20.0	2.0	22.4	22.4	0	0.8000
1.28	25.0	2.5	26.9	26.9	0	0.6937
1.25	30.0	3.0	31.6	31.6	0	0.6250
1.22	35.0	3.5	36.4	36.4	0	0.5750
1.21	40.0	4.0	41.2	41.2	0	0.5437
1.18	50.0	5.0	51.0	51.0	0	0.5104
1.17	60.0	6.0	60.8	60.8	0	0.5062
1.15	70.0	7.0	70.7	70.7	0	0.5062
1.15	80.0	8.0	80.6	80.6	0	0.5021
1.14	90.0	9.0	90.6	90.6	0	0.5021
1.39	10.0	1.0	10.7	16.9	25	1.0000
1.36	15.0	1.5	14.1	21.3	25	0.9427
1.32	20.0	2.0	18.2	25.9	25	0.8698
1.29	25.0	2.5	22.7	30.6	25	0.7781
1.27	30.0	3.0	27.3	35.4	25	0.7240
1.24	35.0	3.5	32.1	40.3	25	0.6719
1.22	40.0	4.0	36.9	45.1	25	0.6406
1.19	50.0	5.0	46.7	55.0	25	0.5865
1.17	60.0	6.0	56.5	64.9	25	0.5687
1.16	70.0	7.0	66.4	74.8	25	0.5562
1.15	80.0	8.0	76.3	84.7	25	0.5542
1.14	90.0	9.0	86.3	94.7	25	0.5542

AVG.NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEEDS UPLEG	DOWNLEG	ROUTE ANGLE	PROBABILITY OF DETECTION
1.39	10.0	1.0	10.0	17.3	30	1.0000
1.36	15.0	1.5	13.2	21.8	30	0.9469
1.32	20.0	2.0	17.3	26.5	30	0.8833
1.28	25.0	2.5	21.8	31.2	30	0.8187
1.26	30.0	3.0	26.5	36.1	30	0.7615
1.25	35.0	3.5	31.2	40.9	30	0.7229
1.23	40.0	4.0	36.1	45.8	30	0.6865
1.20	50.0	5.0	45.8	55.7	30	0.6344
1.18	60.0	6.0	55.7	65.6	30	0.6052
1.16	70.0	7.0	65.6	75.5	30	0.5906
1.15	80.0	8.0	75.5	85.4	30	0.5802
1.15	90.0	9.0	85.4	95.4	30	0.5792
1.39	10.0	1.0	9.2	17.7	35	1.0000
1.35	15.0	1.5	12.4	22.3	35	0.9552
1.31	20.0	2.0	16.4	27.0	35	0.8969
1.28	25.0	2.5	20.9	31.8	35	0.8583
1.26	30.0	3.0	25.6	36.7	35	0.8083
1.24	35.0	3.5	30.4	41.6	35	0.7677
1.23	40.0	4.0	35.2	46.5	35	0.7396
1.20	50.0	5.0	45.0	56.3	35	0.6948
1.18	60.0	6.0	54.9	66.2	35	0.6583
1.17	70.0	7.0	64.8	76.2	35	0.6375
1.16	80.0	8.0	74.7	86.1	35	0.6250
1.15	90.0	9.0	84.7	96.1	35	0.6177
1.39	10.0	1.0	8.5	18.1	40	1.0000
1.34	15.0	1.5	11.5	22.8	40	0.9677
1.31	20.0	2.0	15.6	27.5	40	0.9177
1.28	25.0	2.5	20.1	32.3	40	0.8812
1.25	30.0	3.0	24.8	37.2	40	0.8594
1.23	35.0	3.5	29.6	42.1	40	0.8323
1.22	40.0	4.0	34.4	47.1	40	0.8021
1.20	50.0	5.0	44.2	56.9	40	0.7573
1.19	60.0	6.0	54.1	66.9	40	0.7323
1.17	70.0	7.0	64.0	76.8	40	0.7052
1.16	80.0	8.0	74.0	86.8	40	0.6906
1.15	90.0	9.0	83.9	96.7	40	0.6750

AVG.NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEEDS UPLEG	DOWNLEG	ROUTE ANGLE	PROBABILITY OF DETECTION
1.39	10.0	1.0	7.7	18.5	45	1.0000
1.34	15.0	1.5	10.6	23.2	45	0.9865
1.30	20.0	2.0	14.7	28.0	45	0.9365
1.27	25.0	2.5	19.3	32.8	45	0.9115
1.25	30.0	3.0	24.0	37.7	45	0.8927
1.23	35.0	3.5	28.8	42.7	45	0.8802
1.21	40.0	4.0	33.7	47.6	45	0.8708
1.19	50.0	5.0	43.5	57.5	45	0.8344
1.18	60.0	6.0	53.4	67.4	45	0.8073
1.17	70.0	7.0	63.3	77.4	45	0.7875
1.16	80.0	8.0	73.3	87.4	45	0.7740
1.16	90.0	9.0	83.2	97.3	45	0.7625
1.39	10.0	1.0	6.8	18.8	50	1.0000
1.33	15.0	1.5	9.8	23.6	50	1.0000
1.28	20.0	2.0	13.9	28.4	50	0.9656
1.25	25.0	2.5	18.5	33.3	50	0.9427
1.23	30.0	3.0	23.2	38.2	50	0.9302
1.21	35.0	3.5	28.1	43.1	50	0.9240
1.20	40.0	4.0	33.0	48.1	50	0.9177
1.18	50.0	5.0	42.8	58.0	50	0.9083
1.17	60.0	6.0	52.7	68.0	50	0.9021
1.16	70.0	7.0	62.7	77.9	50	0.8854
1.15	80.0	8.0	72.6	87.9	50	0.8698
1.14	90.0	9.0	82.6	97.9	50	0.8583
1.39	10.0	1.0	6.0	19.1	55	1.0000
1.31	15.0	1.5	8.9	23.9	55	1.0000
1.26	20.0	2.0	13.1	28.8	55	1.0000
1.22	25.0	2.5	17.8	33.7	55	0.9844
1.20	30.0	3.0	22.6	38.6	55	0.9719
1.18	35.0	3.5	27.4	43.6	55	0.9656
1.17	40.0	4.0	32.3	48.5	55	0.9615
1.15	50.0	5.0	42.2	58.5	55	0.9552
1.13	60.0	6.0	52.1	68.4	55	0.9510
1.12	70.0	7.0	62.1	78.4	55	0.9490
1.12	80.0	8.0	72.0	88.4	55	0.9469
1.11	90.0	9.0	82.0	98.4	55	0.9448

AVG.NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEEDS UPLEG	DOWNLEG	ROUTE ANGLE	PROBABILITY OF DETECTION
1.39	10.0	1.0	5.2	19.3	60	1.0000
1.29	15.0	1.5	8.1	24.2	60	1.0000
1.23	20.0	2.0	12.4	29.1	60	1.0000
1.19	25.0	2.5	17.1	34.0	60	1.0000
1.16	30.0	3.0	21.9	39.0	60	1.0000
1.14	35.0	3.5	26.8	43.9	60	1.0000
1.12	40.0	4.0	31.7	48.9	60	1.0000
1.10	50.0	5.0	41.6	58.9	60	1.0000
1.08	60.0	6.0	51.6	68.8	60	1.0000
1.07	70.0	7.0	61.5	78.8	60	1.0000
1.06	80.0	8.0	71.5	88.8	60	1.0000
1.05	90.0	9.0	81.5	98.8	60	1.0000

PROBABILITY OF RANDOM ENCOUNTER
EXTENDED MODEL

DEFINITE RANGE LAW
DETECTION RANGE: 30.0 MILES

LENGTH OF SUB PATROL LINE: 120.0 MILES

X-DISTANCE TO PATROL LINE: 30.0 MILES

CROSSING ANGLE: 0.0 DEGREES (PERPENDICULAR)

SUBMARINE SPEED: 10.0 KNOTS

NO. OF SAMPLES TAKEN FOR EACH SCENARIO: 960

AVG.NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEEDS UPLEG	DOWNLEG	ROUTE POSIT	PROBABILITY OF DETECTION
1.40	10.0	1.0	14.1	14.1	60.0	1.0000
1.36	15.0	1.5	18.0	18.0	60.0	0.9354
1.33	20.0	2.0	22.4	22.4	60.0	0.8000
1.28	25.0	2.5	26.9	26.9	60.0	0.6937
1.25	30.0	3.0	31.6	31.6	60.0	0.6250
1.21	40.0	4.0	41.2	41.2	60.0	0.5437
1.18	50.0	5.0	51.0	51.0	60.0	0.5104
2.32	90.0	9.0	90.6	90.6	60.0	0.5021
1.40	10.0	1.0	14.1	14.1	65.0	1.0000
1.36	15.0	1.5	18.0	18.0	65.0	0.9260
1.32	20.0	2.0	22.4	22.4	65.0	0.7844
1.28	25.0	2.5	26.9	26.9	65.0	0.6937
1.25	30.0	3.0	31.6	31.6	65.0	0.6250
1.21	40.0	4.0	41.2	41.2	65.0	0.5500
1.18	50.0	5.0	51.0	51.0	65.0	0.5260
2.32	90.0	9.0	90.6	90.6	65.0	0.5021
1.40	10.0	1.0	14.1	14.1	70.0	1.0000
1.35	15.0	1.5	18.0	18.0	70.0	0.8687
1.30	20.0	2.0	22.4	22.4	70.0	0.7635
1.26	25.0	2.5	26.9	26.9	70.0	0.6750
1.24	30.0	3.0	31.6	31.6	70.0	0.6177
1.21	40.0	4.0	41.2	41.2	70.0	0.5708
1.18	50.0	5.0	51.0	51.0	70.0	0.5469
2.32	90.0	9.0	90.6	90.6	70.0	0.5021
1.40	10.0	1.0	14.1	14.1	75.0	0.9781
1.33	15.0	1.5	18.0	18.0	75.0	0.8062
1.28	20.0	2.0	22.4	22.4	75.0	0.7125
1.24	25.0	2.5	26.9	26.9	75.0	0.6542
1.22	30.0	3.0	31.6	31.6	75.0	0.6104
1.19	40.0	4.0	41.2	41.2	75.0	0.5771
1.18	50.0	5.0	51.0	51.0	75.0	0.5604
2.31	90.0	9.0	90.6	90.6	75.0	0.5229

AVG.NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEEDS UPLEG	DOWNLEG	ROUTE POSIT	PROBABILITY OF DETECTION
1.40	10.0	1.0	14.1	14.1	80.0	0.9365
1.31	15.0	1.5	18.0	18.0	80.0	0.7437
1.26	20.0	2.0	22.4	22.4	80.0	0.6500
1.22	25.0	2.5	26.9	26.9	80.0	0.6031
1.20	30.0	3.0	31.6	31.6	80.0	0.5969
1.17	40.0	4.0	41.2	41.2	80.0	0.5771
1.16	50.0	5.0	51.0	51.0	80.0	0.5604
2.28	90.0	9.0	90.6	90.6	80.0	0.5292
1.39	10.0	1.0	14.1	14.1	85.0	0.8844
1.29	15.0	1.5	18.0	18.0	85.0	0.6812
1.23	20.0	2.0	22.4	22.4	85.0	0.5875
1.20	25.0	2.5	26.9	26.9	85.0	0.5615
1.18	30.0	3.0	31.6	31.6	85.0	0.5552
1.15	40.0	4.0	41.2	41.2	85.0	0.5490
1.13	50.0	5.0	51.0	51.0	85.0	0.5469
2.24	90.0	9.0	90.6	90.6	85.0	0.5292
1.36	10.0	1.0	14.1	14.1	90.0	0.8219
1.27	15.0	1.5	18.0	18.0	90.0	0.6187
1.21	20.0	2.0	22.4	22.4	90.0	0.5302
1.18	25.0	2.5	26.9	26.9	90.0	0.5198
1.16	30.0	3.0	31.6	31.6	90.0	0.5135
1.13	40.0	4.0	41.2	41.2	90.0	0.5073
1.11	50.0	5.0	51.0	51.0	90.0	0.5052
2.19	90.0	9.0	90.6	90.6	90.0	0.5010
1.30	10.0	1.0	14.1	14.1	110.0	0.5719
1.21	15.0	1.5	18.0	18.0	110.0	0.3844
1.17	20.0	2.0	22.4	22.4	110.0	0.3635
1.14	25.0	2.5	26.9	26.9	110.0	0.3531
1.13	30.0	3.0	31.6	31.6	110.0	0.3469
1.11	40.0	4.0	41.2	41.2	110.0	0.3406
1.09	50.0	5.0	51.0	51.0	110.0	0.3385
2.16	90.0	9.0	90.6	90.6	110.0	0.3344
1.33	10.0	1.0	14.1	14.1	130.0	0.3219
1.23	15.0	1.5	18.0	18.0	130.0	0.2177
1.18	20.0	2.0	22.4	22.4	130.0	0.1969
1.14	25.0	2.5	26.9	26.9	130.0	0.1865
1.13	30.0	3.0	31.6	31.6	130.0	0.1802
1.11	40.0	4.0	41.2	41.2	130.0	0.1740
1.09	50.0	5.0	51.0	51.0	130.0	0.1719
2.16	90.0	9.0	90.6	90.6	130.0	0.1677
1.58	10.0	1.0	14.1	14.1	150.0	0.0
1.42	15.0	1.5	18.0	18.0	150.0	0.0
1.33	20.0	2.0	22.4	22.4	150.0	0.0010
1.28	25.0	2.5	26.9	26.9	150.0	0.0010
1.25	30.0	3.0	31.6	31.6	150.0	0.0010
1.21	40.0	4.0	41.2	41.2	150.0	0.0010
1.18	50.0	5.0	51.0	51.0	150.0	0.0010
2.32	90.0	9.0	90.6	90.6	150.0	0.0

MONTE CARLO TECHNIQUE
PROBABILITY OF RANDOM ENCOUNTER
MID-POINT CROSSING

DEFINITE RANGE LAW
DETECTION RANGE: 30.0 MILES

LENGTH OF SUB PATROL LINE: 120.0 MILES

X-DISTANCE TO PATROL LINE: 30.0 MILES

SUBMARINE SPEED: 10.0 KNOTS

NO. OF SAMPLES TAKEN FOR EACH SCENARIO: 1000

AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	RELATIVE SPEED DOWNLEG	ALFA	PROB. DETECTION
1.41	10.0	1.0	14.1	14.1	0.0	1.0000
1.34	20.0	2.0	22.4	22.4	0.0	0.8200
1.25	30.0	3.0	31.6	31.6	0.0	0.6270
1.21	40.0	4.0	41.2	41.2	0.0	0.5470
1.18	50.0	5.0	51.0	51.0	0.0	0.5290
1.17	60.0	6.0	60.8	60.8	0.0	0.5060
1.15	70.0	7.0	70.7	70.7	0.0	0.4810
1.16	80.0	8.0	80.6	80.6	0.0	0.4810
1.13	90.0	9.0	90.6	90.6	0.0	0.5070
1.42	10.0	1.0	13.5	14.7	0.0873	1.0000
1.32	20.0	2.0	21.6	23.1	0.0873	0.7650
1.25	30.0	3.0	30.8	32.4	0.0873	0.6420
1.19	40.0	4.0	40.4	42.1	0.0873	0.5460
1.18	50.0	5.0	50.1	51.8	0.0873	0.5210
1.19	60.0	6.0	60.0	61.7	0.0873	0.5020
1.15	70.0	7.0	69.8	71.6	0.0873	0.4980
1.14	80.0	8.0	79.8	81.5	0.0873	0.5180
1.14	90.0	9.0	89.7	91.4	0.0873	0.4930
1.38	10.0	1.0	12.9	15.3	0.1745	1.0000
1.34	20.0	2.0	20.7	23.9	0.1745	0.8140
1.24	30.0	3.0	29.9	33.2	0.1745	0.6380
1.23	40.0	4.0	39.5	42.9	0.1745	0.5640
1.17	50.0	5.0	49.3	52.7	0.1745	0.5190
1.15	60.0	6.0	59.1	62.5	0.1745	0.5130
1.15	70.0	7.0	69.0	72.4	0.1745	0.5210
1.14	80.0	8.0	78.9	82.3	0.1745	0.5070
1.14	90.0	9.0	88.8	92.3	0.1745	0.5000

AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	DOWNLEG	ALFA	PROB. DETECTION
1.40	10.0	1.0	12.2	15.9	0.2618	1.0000
1.32	20.0	2.0	19.9	24.6	0.2618	0.8120
1.26	30.0	3.0	29.1	34.0	0.2618	0.6500
1.22	40.0	4.0	38.6	43.7	0.2618	0.5530
1.20	50.0	5.0	48.4	53.5	0.2618	0.5260
1.17	60.0	6.0	58.2	63.3	0.2618	0.5270
1.15	70.0	7.0	68.1	73.2	0.2618	0.5190
1.14	80.0	8.0	78.0	83.2	0.2618	0.5150
1.15	90.0	9.0	87.9	93.1	0.2618	0.5190
1.39	10.0	1.0	11.5	16.4	0.3491	1.0000
1.32	20.0	2.0	19.1	25.2	0.3491	0.8600
1.24	30.0	3.0	28.2	34.7	0.3491	0.6920
1.20	40.0	4.0	37.8	44.4	0.3491	0.6070
1.19	50.0	5.0	47.5	54.2	0.3491	0.5470
1.18	60.0	6.0	57.4	64.1	0.3491	0.5480
1.16	70.0	7.0	67.2	74.0	0.3491	0.5290
1.14	80.0	8.0	77.2	83.9	0.3491	0.5340
1.16	90.0	9.0	87.1	93.9	0.3491	0.5070
1.42	10.0	1.0	10.7	16.9	0.4363	1.0000
1.30	20.0	2.0	18.2	25.9	0.4363	0.8570
1.26	30.0	3.0	27.3	35.4	0.4363	0.7380
1.22	40.0	4.0	36.9	45.1	0.4363	0.6430
1.18	50.0	5.0	46.7	55.0	0.4363	0.5910
1.18	60.0	6.0	56.5	64.9	0.4363	0.5530
1.15	70.0	7.0	66.4	74.8	0.4363	0.5860
1.16	80.0	8.0	76.3	84.7	0.4363	0.5360
1.13	90.0	9.0	86.3	94.7	0.4363	0.5490
1.41	10.0	1.0	10.0	17.3	0.5236	1.0000
1.30	20.0	2.0	17.3	26.5	0.5236	0.8890
1.26	30.0	3.0	26.5	36.1	0.5236	0.7270
1.24	40.0	4.0	36.1	45.8	0.5236	0.6690
1.21	50.0	5.0	45.8	55.7	0.5236	0.6100
1.18	60.0	6.0	55.7	65.6	0.5236	0.5830
1.18	70.0	7.0	65.6	75.5	0.5236	0.5910
1.15	80.0	8.0	75.5	85.4	0.5236	0.5780
1.13	90.0	9.0	85.4	95.4	0.5236	0.5540
1.37	10.0	1.0	9.2	17.7	0.6109	1.0000
1.31	20.0	2.0	16.4	27.0	0.6109	0.9010
1.26	30.0	3.0	25.6	36.7	0.6109	0.8140
1.25	40.0	4.0	35.2	46.5	0.6109	0.7480
1.19	50.0	5.0	45.0	56.3	0.6109	0.7090
1.19	60.0	6.0	54.9	66.2	0.6109	0.6540
1.17	70.0	7.0	64.8	76.2	0.6109	0.6190
1.16	80.0	8.0	74.7	86.1	0.6109	0.6290
1.14	90.0	9.0	84.7	96.1	0.6109	0.6310

AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	DOWNLEG	ALFA	PROB. DETECTION
1.40	10.0	1.0	8.5	18.1	0.6981	1.0000
1.31	20.0	2.0	15.6	27.5	0.6981	0.9350
1.24	30.0	3.0	24.8	37.2	0.6981	0.8720
1.23	40.0	4.0	34.4	47.1	0.6981	0.8090
1.22	50.0	5.0	44.2	56.9	0.6981	0.7590
1.19	60.0	6.0	54.1	66.9	0.6981	0.7560
1.16	70.0	7.0	64.0	76.8	0.6981	0.7080
1.16	80.0	8.0	74.0	86.8	0.6981	0.7080
1.14	90.0	9.0	83.9	96.7	0.6981	0.6880
1.42	10.0	1.0	7.7	18.5	0.7854	1.0000
1.30	20.0	2.0	14.7	28.0	0.7854	0.9410
1.25	30.0	3.0	24.0	37.7	0.7854	0.8740
1.20	40.0	4.0	33.7	47.6	0.7854	0.8550
1.20	50.0	5.0	43.5	57.5	0.7854	0.8310
1.20	60.0	6.0	53.4	67.4	0.7854	0.7850
1.16	70.0	7.0	63.3	77.4	0.7854	0.8060
1.16	80.0	8.0	73.3	87.4	0.7854	0.7680
1.17	90.0	9.0	83.2	97.3	0.7854	0.7560
1.38	10.0	1.0	6.8	18.8	0.8727	1.0000
1.26	20.0	2.0	13.9	28.4	0.8727	0.9570
1.24	30.0	3.0	23.2	38.2	0.8727	0.9260
1.22	40.0	4.0	33.0	48.1	0.8727	0.9130
1.17	50.0	5.0	42.8	58.0	0.8727	0.9140
1.17	60.0	6.0	52.7	68.0	0.8727	0.8940
1.16	70.0	7.0	62.7	77.9	0.8727	0.8830
1.14	80.0	8.0	72.6	87.9	0.8727	0.8650
1.14	90.0	9.0	82.6	97.9	0.8727	0.8410
1.39	10.0	1.0	6.0	19.1	0.9599	1.0000
1.25	20.0	2.0	13.1	28.8	0.9599	1.0000
1.18	30.0	3.0	22.6	38.6	0.9599	0.9740
1.16	40.0	4.0	32.3	48.5	0.9599	0.9620
1.16	50.0	5.0	42.2	58.5	0.9599	0.9520
1.12	60.0	6.0	52.1	68.4	0.9599	0.9470
1.12	70.0	7.0	62.1	78.4	0.9599	0.9420
1.11	80.0	8.0	72.0	88.4	0.9599	0.9430
1.11	90.0	9.0	82.0	98.4	0.9599	0.9410
1.38	10.0	1.0	5.2	19.3	1.0472	1.0000
1.23	20.0	2.0	12.4	29.1	1.0472	1.0000
1.16	30.0	3.0	21.9	39.0	1.0472	1.0000
1.12	40.0	4.0	31.7	48.9	1.0472	1.0000
1.08	50.0	5.0	41.6	58.9	1.0472	1.0000
1.10	60.0	6.0	51.6	68.8	1.0472	1.0000
1.08	70.0	7.0	61.5	78.7	1.0472	1.0000
1.07	80.0	8.0	71.5	88.8	1.0472	1.0000
1.05	90.0	9.0	81.5	98.8	1.0472	1.0000

AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	RELATIVE SPEED DOWNLEG	ALFA	PROB. DETECTION
1.44	10.0	1.0	4.3	19.5	1.1345	1.0000
1.24	20.0	2.0	11.7	29.4	1.1345	1.0000
1.15	30.0	3.0	21.4	39.3	1.1345	1.0000
1.12	40.0	4.0	31.2	49.2	1.1345	1.0000
1.09	50.0	5.0	41.2	59.2	1.1345	1.0000
1.08	60.0	6.0	51.1	69.2	1.1345	1.0000
1.07	70.0	7.0	61.1	79.2	1.1345	1.0000
1.05	80.0	8.0	71.1	89.2	1.1345	1.0000
1.04	90.0	9.0	81.0	99.2	1.1345	1.0000
1.55	10.0	1.0	3.5	19.7	1.2217	1.0000
1.27	20.0	2.0	11.1	29.6	1.2217	1.0000
1.18	30.0	3.0	20.9	39.5	1.2217	1.0000
1.15	40.0	4.0	30.8	49.5	1.2217	1.0000
1.11	50.0	5.0	40.7	59.5	1.2217	1.0000
1.11	60.0	6.0	50.7	69.5	1.2217	1.0000
1.08	70.0	7.0	60.7	79.5	1.2217	1.0000
1.07	80.0	8.0	70.7	89.5	1.2217	1.0000
1.07	90.0	9.0	80.7	99.5	1.2217	1.0000
1.76	10.0	1.0	2.6	19.8	1.3090	1.0000
1.36	20.0	2.0	10.7	29.8	1.3090	1.0000
1.25	30.0	3.0	20.5	39.7	1.3090	1.0000
1.18	40.0	4.0	30.5	49.7	1.3090	1.0000
1.14	50.0	5.0	40.4	59.7	1.3090	1.0000
1.12	60.0	6.0	50.4	69.7	1.3090	1.0000
1.12	70.0	7.0	60.4	79.7	1.3090	1.0000
1.08	80.0	8.0	70.4	89.7	1.3090	1.0000
1.08	90.0	9.0	80.4	99.7	1.3090	1.0000
2.19	10.0	1.0	1.7	19.9	1.3963	1.0000
1.63	20.0	2.0	10.3	29.9	1.3963	1.0000
1.44	30.0	3.0	20.2	39.9	1.3963	1.0000
1.28	40.0	4.0	30.2	49.9	1.3963	1.0000
1.27	50.0	5.0	40.2	59.9	1.3963	1.0000
1.18	60.0	6.0	50.2	69.9	1.3963	1.0000
1.15	70.0	7.0	60.2	79.9	1.3963	1.0000
1.15	80.0	8.0	70.2	89.9	1.3693	1.0000
1.15	90.0	9.0	80.2	99.9	1.3963	1.0000

MONTE CARLO TECHNIQUE
PROBABILITY OF RANDOM ENCOUNTER
NORMALIZED CONVOY ROUTE

DEFINITE RANGE LAW
DETECTION RANGE: 30.0 MILES

LENGTH OF SUB PATROL LINE: 120.0 MILES

X-DISTANCE TO PATROL LINE: 30.0 MILES

SUBMARINE SPEED: 10.0 KNOTS

NO. OF SAMPLES TAKEN FOR EACH SCENARIO: 1000

AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	RELATIVE SPEED DOWNLEG	ALFA	PROB. DETECTION
1.39	10.0	1.0	14.1	14.1	0.0	0.9212
1.27	20.0	2.0	22.4	22.4	0.0	0.6714
1.20	30.0	3.0	31.6	31.6	0.0	0.5720
1.19	40.0	4.0	41.2	41.2	0.0	0.5512
1.16	50.0	5.0	51.0	51.0	0.0	0.5324
1.16	60.0	6.0	60.8	60.8	0.0	0.5221
1.12	70.0	7.0	70.7	70.7	0.0	0.5000
1.13	80.0	8.0	80.6	80.6	0.0	0.5181
1.12	90.0	9.0	90.6	90.6	0.0	0.5133

PROBABILITY OF RANDOM ENCOUNTER
EXTENDED MODEL

DEFINITE RANGE LAW
DETECTION RANGE: 30.0 MILES

LENGTH OF SUB PATROL LINE: 90.0 MILES

X-DISTANCE TO PATROL LINE: 30.0 MILES

CROSSING ANGLE: 0.0 DEGREES (PERPENDICULAR)

SUBMARINE SPEED: 10.0 KNOTS

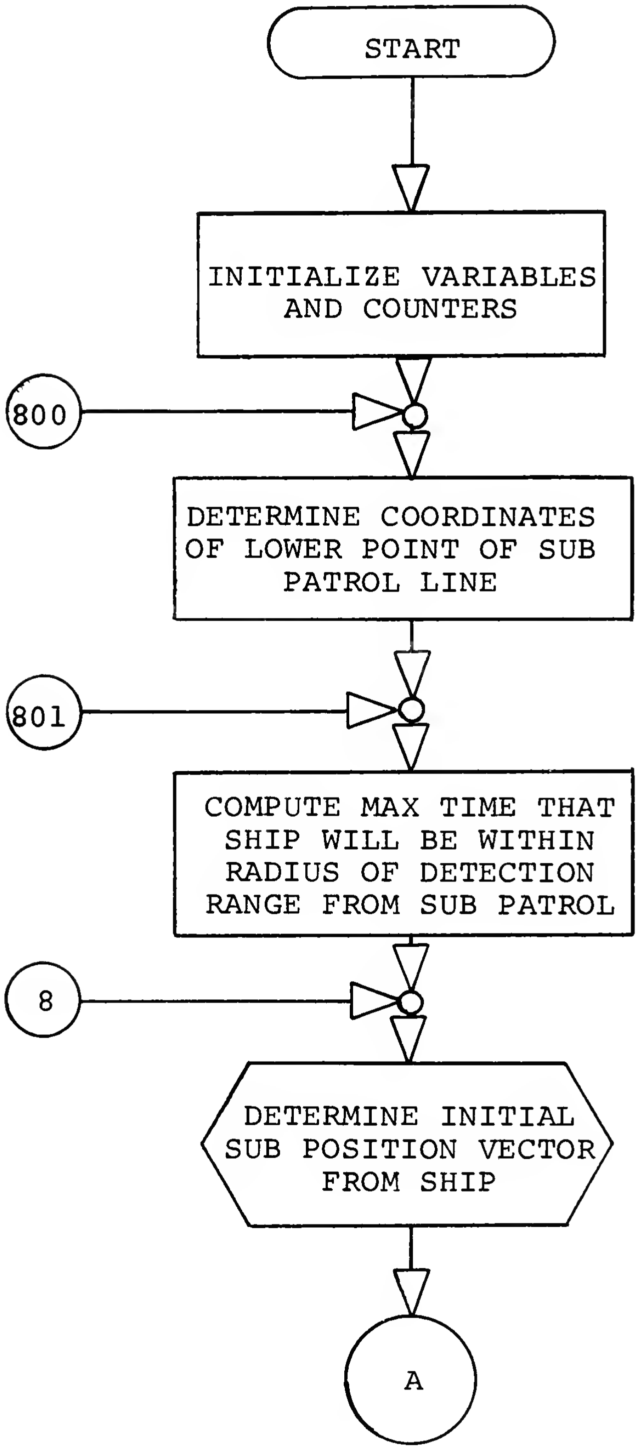
NO. OF SAMPLES TAKEN FOR EACH SCENARIO: 720

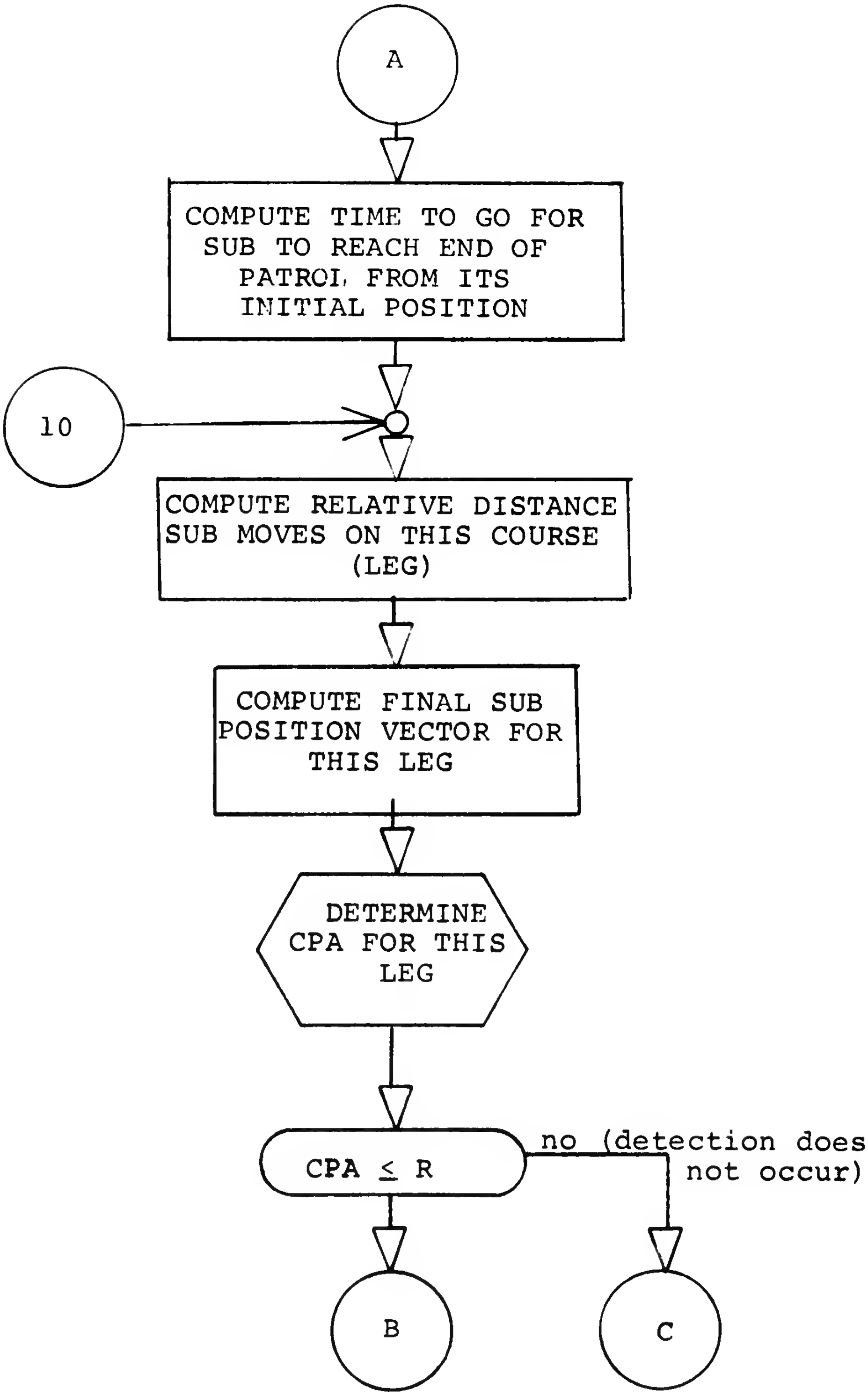
AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	DOWNLEG	CROSS POSIT	PROBABILITY OF DETECTION
1.36	10.0	1.0	14.1	14.1	45.0	1.0000
1.32	15.0	1.5	18.0	18.0	45.0	1.0000
1.29	20.0	2.0	22.4	22.4	45.0	1.0000
1.27	25.0	2.5	26.9	26.9	45.0	0.9861
1.26	30.0	3.0	31.6	31.6	45.0	0.9250
1.24	40.0	4.0	41.2	41.2	45.0	0.8528
1.22	50.0	5.0	51.0	51.0	45.0	0.8139
2.41	90.0	9.0	90.6	90.6	45.0	0.7250
1.36	10.0	1.0	14.1	14.1	50.0	1.0000
1.32	15.0	1.5	18.0	18.0	50.0	1.0000
1.29	20.0	2.0	22.4	22.4	50.0	0.9847
1.27	25.0	2.5	26.9	26.9	50.0	0.9375
1.26	30.0	3.0	31.6	31.6	50.0	0.9069
1.23	40.0	4.0	41.2	41.2	50.0	0.8444
1.21	50.0	5.0	51.0	51.0	50.0	0.7958
2.37	90.0	9.0	90.6	90.6	50.0	0.7056
1.36	10.0	1.0	14.1	14.1	55.0	1.0000
1.32	15.0	1.5	18.0	18.0	55.0	1.0000
1.29	20.0	2.0	22.4	22.4	55.0	0.9292
1.27	25.0	2.5	26.9	26.9	55.0	0.8792
1.24	30.0	3.0	31.6	31.6	55.0	0.8333
1.20	40.0	4.0	41.2	41.2	55.0	0.7792
1.18	50.0	5.0	51.0	51.0	55.0	0.7500
2.32	90.0	9.0	90.6	90.6	55.0	0.7056

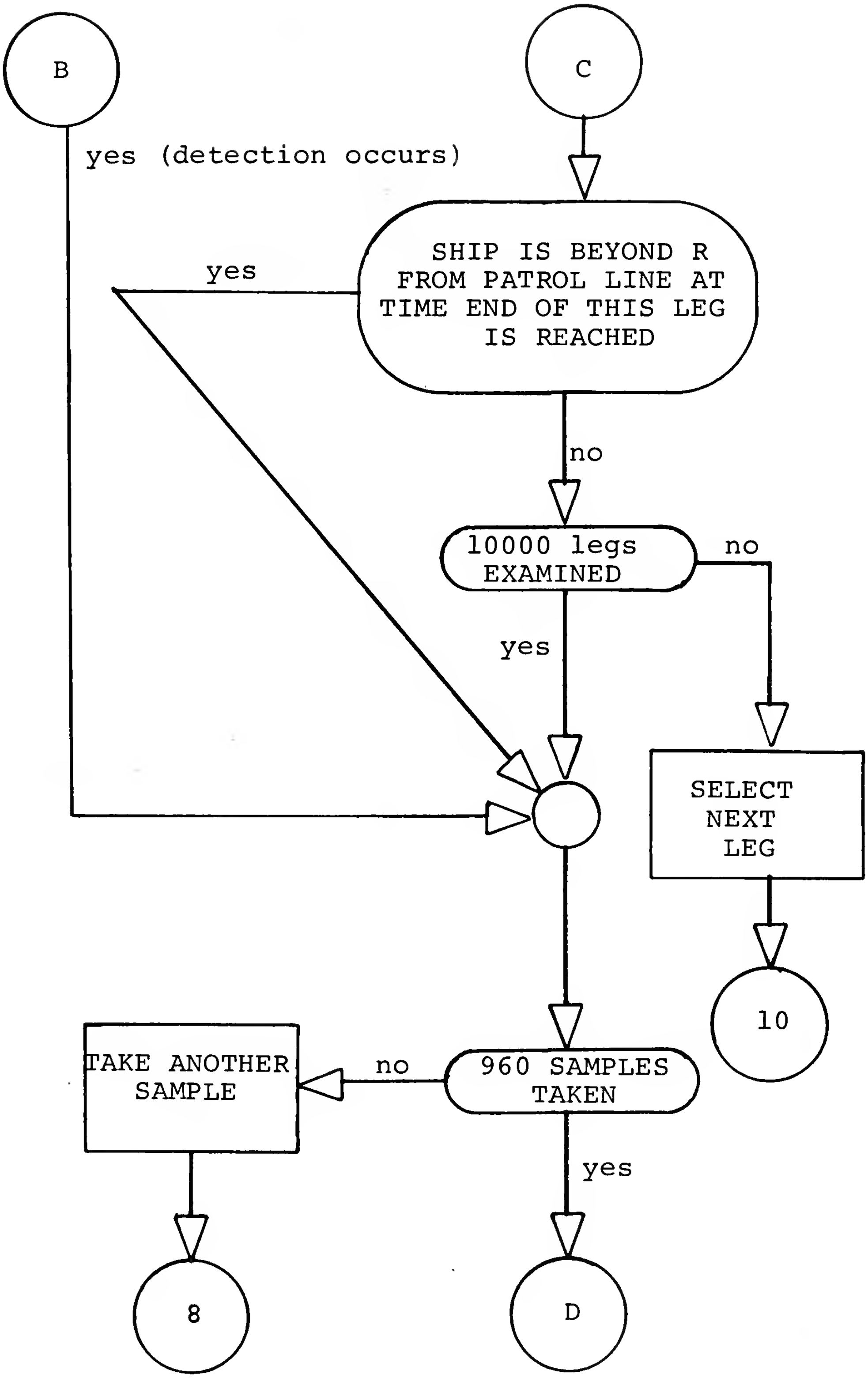
AVG. NO. LEGS	SHIP SPEED	SPEED RATIO	RELATIVE SPEED UPLEG	DOWNLEG	CROSS POSIT	PROBABILITY OF DETECTION
1.36	10.0	1.0	14.1	14.1	60.0	1.0000
1.32	15.0	1.5	18.0	18.0	60.0	0.9569
1.28	20.0	2.0	22.4	22.4	60.0	0.8667
1.24	25.0	2.5	26.9	26.9	60.0	0.7958
1.21	30.0	3.0	31.6	31.6	60.0	0.7500
1.17	40.0	4.0	41.2	41.2	60.0	0.6958
1.15	50.0	5.0	51.0	51.0	60.0	0.6736
2.26	90.0	9.0	90.6	90.6	60.0	0.6681
1.36	10.0	1.0	14.1	14.1	65.0	1.0000
1.32	15.0	1.5	18.0	18.0	65.0	0.9014
1.26	20.0	2.0	22.4	22.4	65.0	0.7833
1.22	25.0	2.5	26.9	26.9	65.0	0.7125
1.19	30.0	3.0	31.6	31.6	65.0	0.6667
1.16	40.0	4.0	41.2	41.2	65.0	0.6208
1.14	50.0	5.0	51.0	51.0	65.0	0.6181
2.23	90.0	9.0	90.6	90.6	65.0	0.6125
1.36	10.0	1.0	14.1	14.1	70.0	1.0000
1.31	15.0	1.5	18.0	18.0	70.0	0.8250
1.24	20.0	2.0	22.4	22.4	70.0	0.7000
1.21	25.0	2.5	26.9	26.9	70.0	0.6292
1.18	30.0	3.0	31.6	31.6	70.0	0.5833
1.15	40.0	4.0	41.2	41.2	70.0	0.5653
1.13	50.0	5.0	51.0	51.0	70.0	0.5625
2.23	90.0	9.0	90.6	90.6	70.0	0.5569

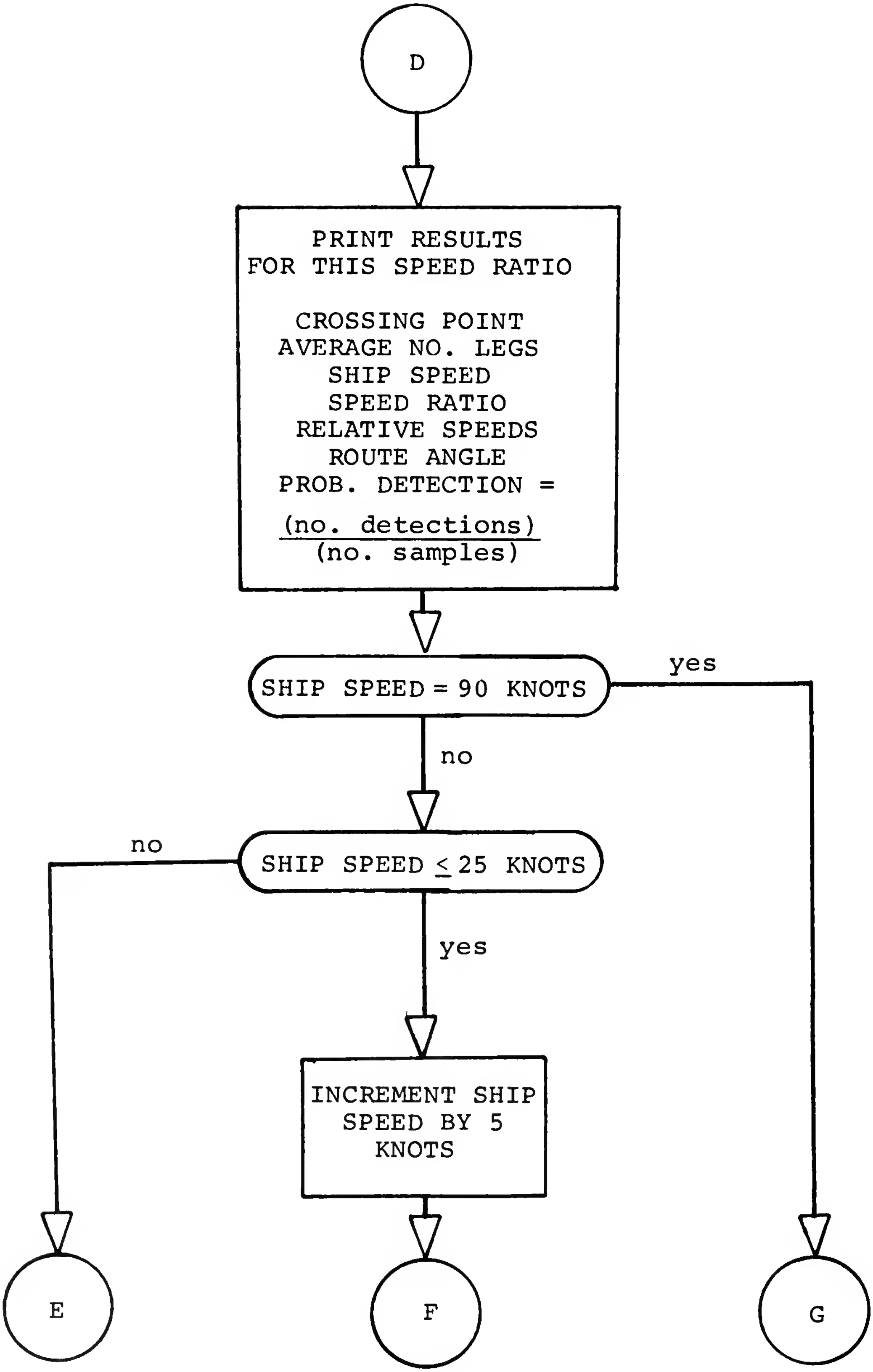
APPENDIX B

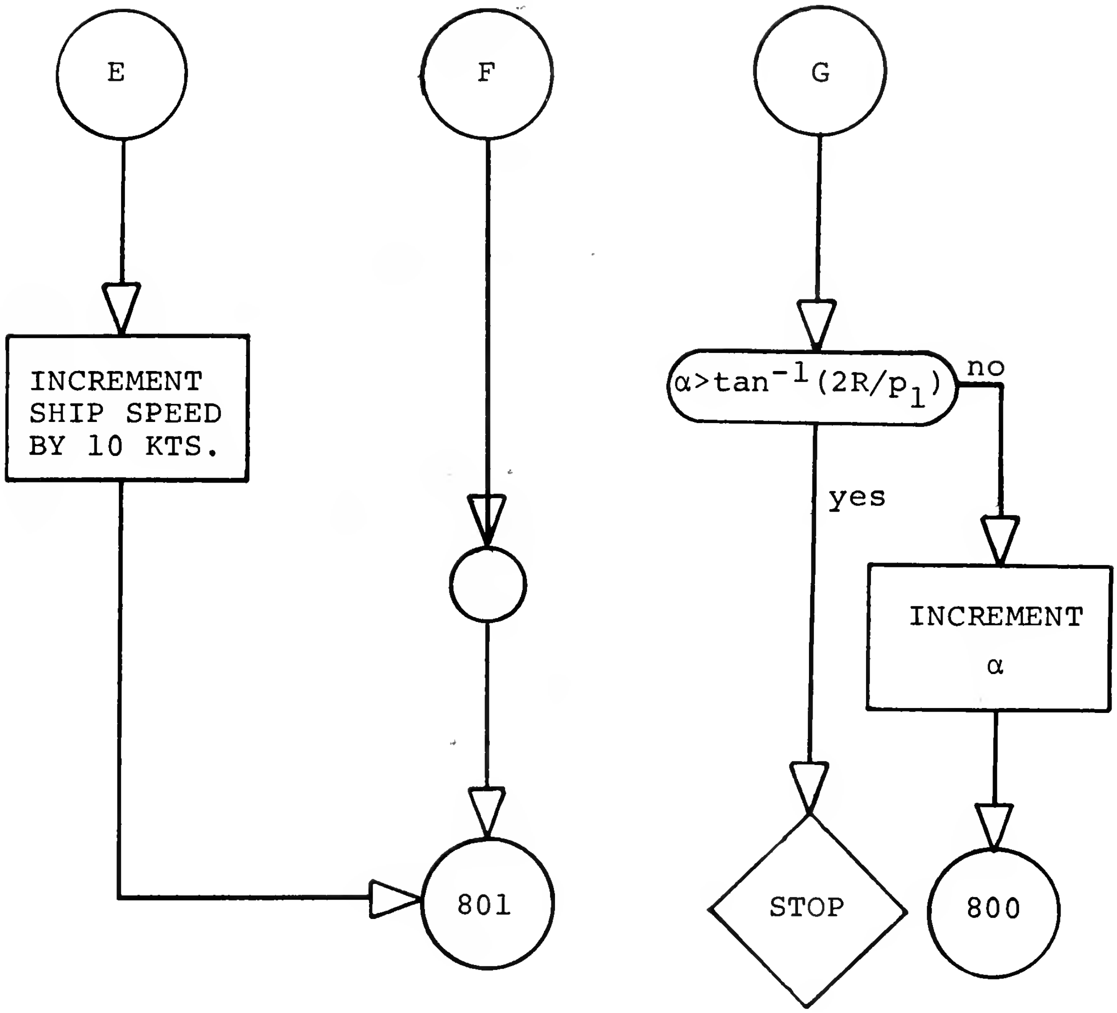
MAIN COMPUTER PROGRAM FLOW DIAGRAM



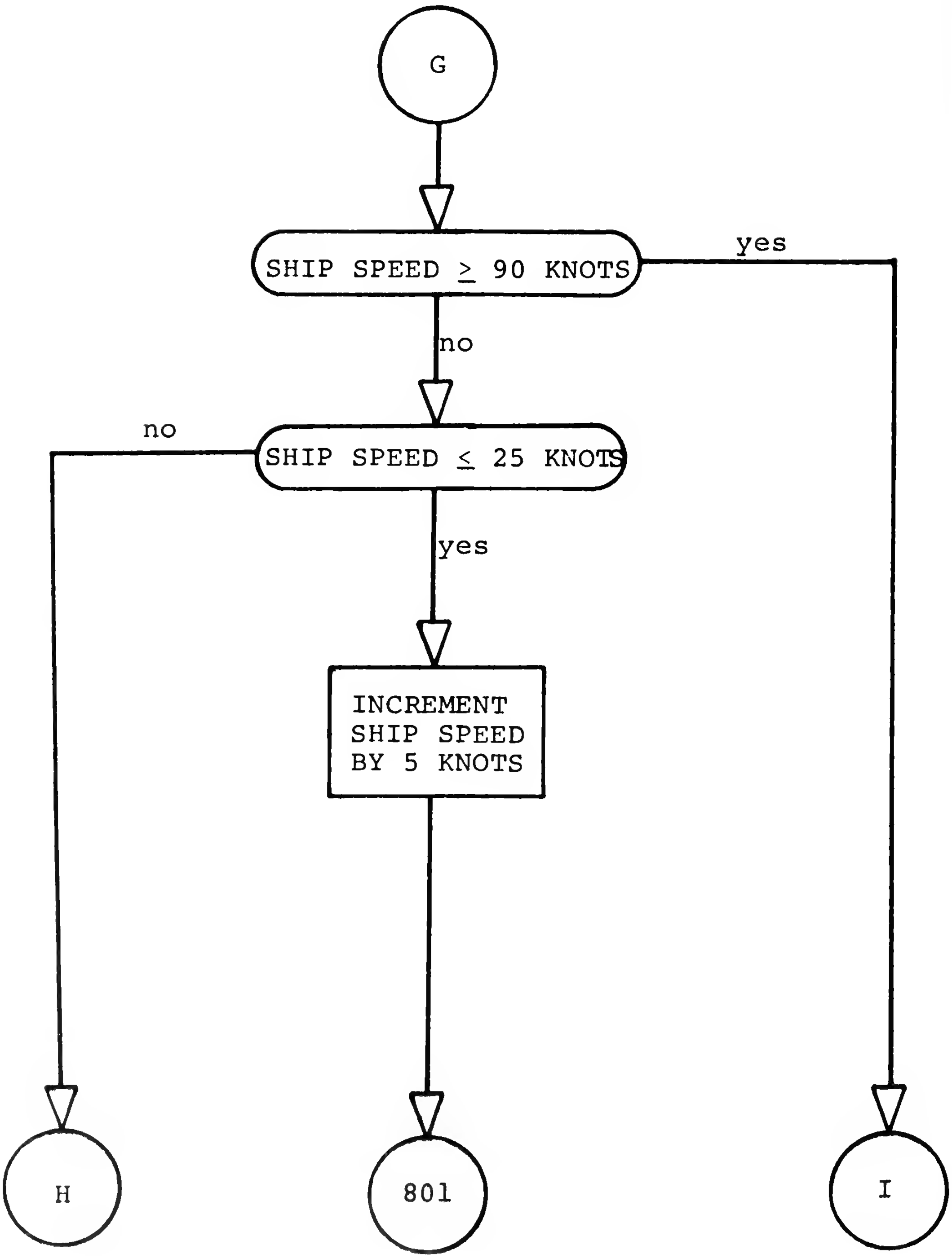

A
COMPUTE TIME TO GO FOR SUB TO REACH END OF PATROL FROM ITS INITIAL POSITION
10
COMPUTE RELATIVE DISTANCE SUB MOVES ON THIS COURSE (LEG)
COMPUTE FINAL SUB POSITION VECTOR FOR THIS LEG
DETERMINE CPA FOR THIS LEG
CPA ≤ R
no (detection does not occur)
B
C


B
C
yes (detection occurs)
SHIP IS BEYOND R FROM PATROL LINE AT TIME END OF THIS LEG IS REACHED
yes
no
10000 legs EXAMINED
no
yes
SELECT NEXT LEG
10
TAKE ANOTHER SAMPLE
no
960 SAMPLES TAKEN
yes
8
D

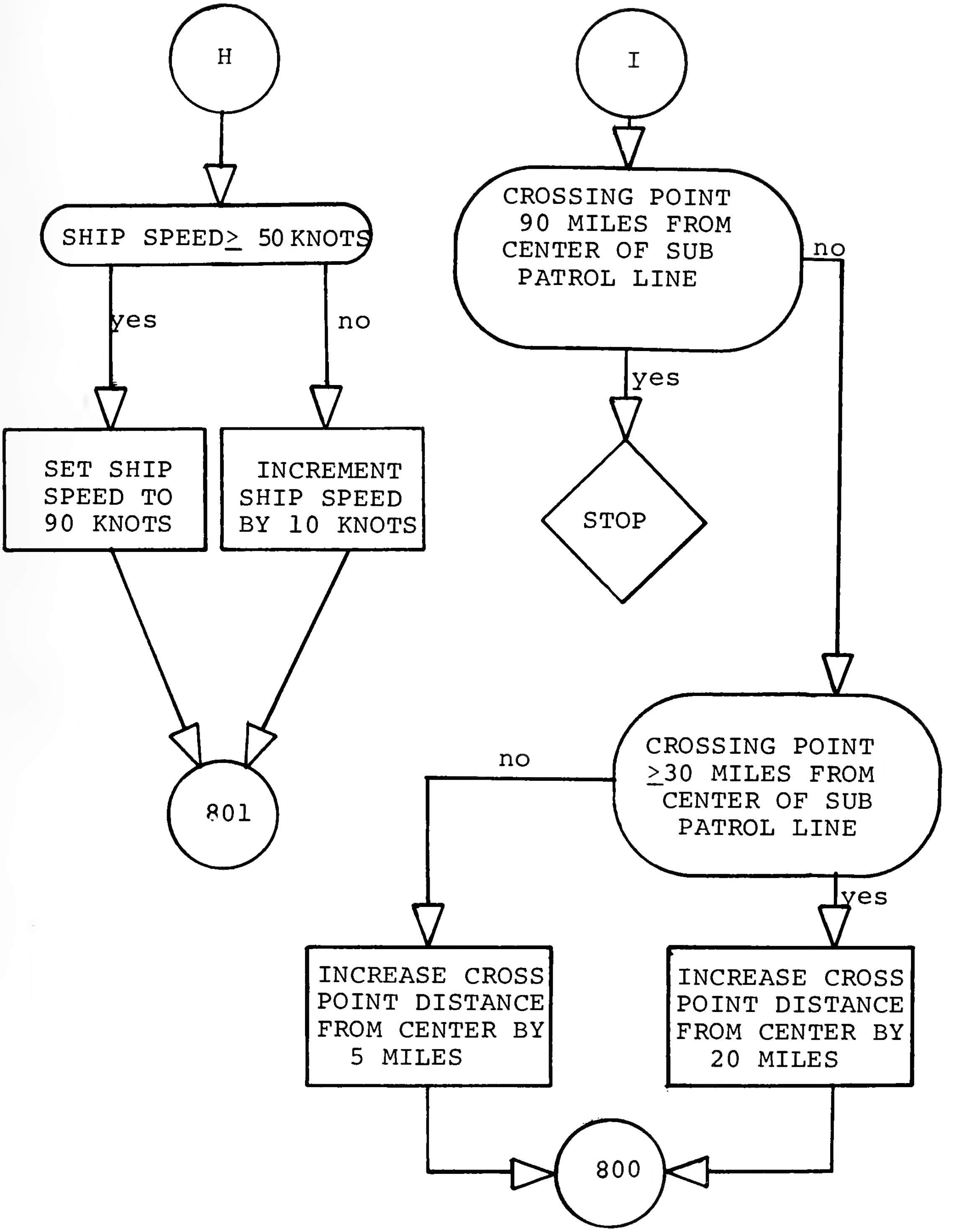

D
PRINT RESULTS FOR THIS SPEED RATIO
CROSSING POINT
AVERAGE NO. LEGS
SHIP SPEED
SPEED RATIO
RELATIVE SPEEDS
ROUTE ANGLE
PROB. DETECTION = $\frac{\text{(no. detections)}}{\text{(no. samples)}}$
SHIP SPEED = 90 KNOTS
yes
no
SHIP SPEED ≤ 25 KNOTS
no
yes
INCREMENT SHIP SPEED BY 5 KNOTS
E
F
G


E
F
G
INCREMENT SHIP SPEED BY 10 KTS.
α>tan⁻¹(2R/p₁)
no
yes
INCREMENT α
801
STOP
800

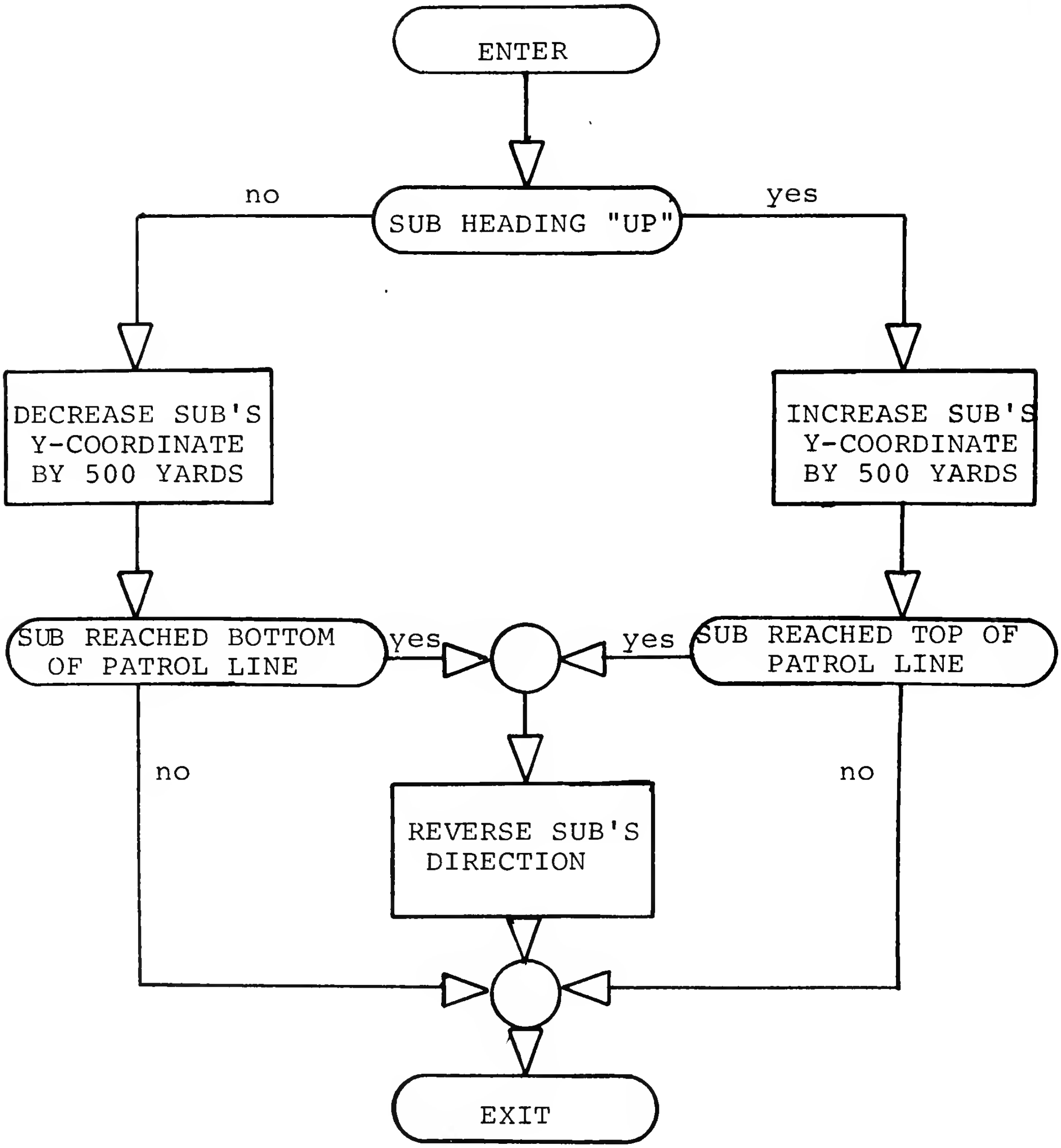
EXTENDED MODEL



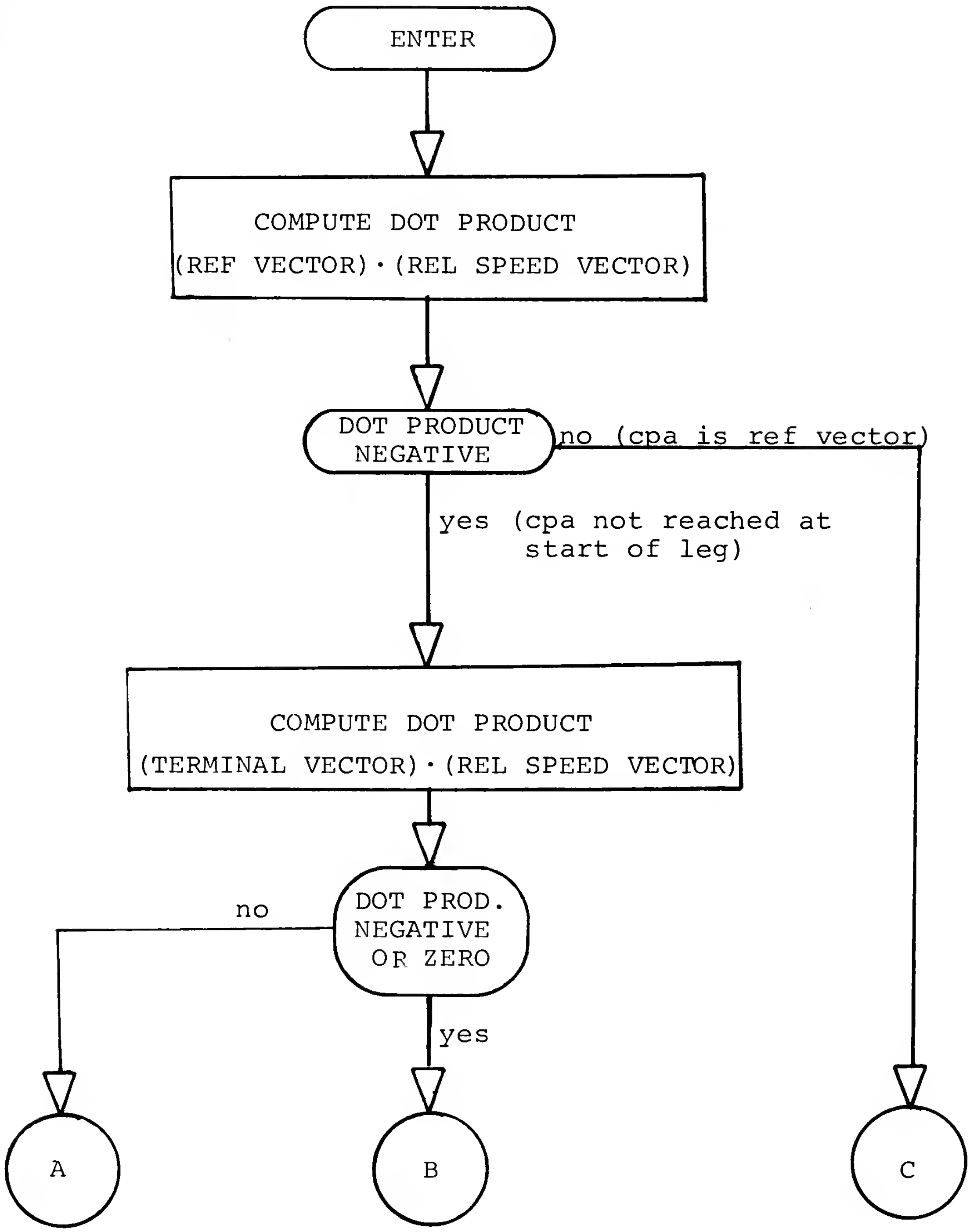
EXTENDED MODEL

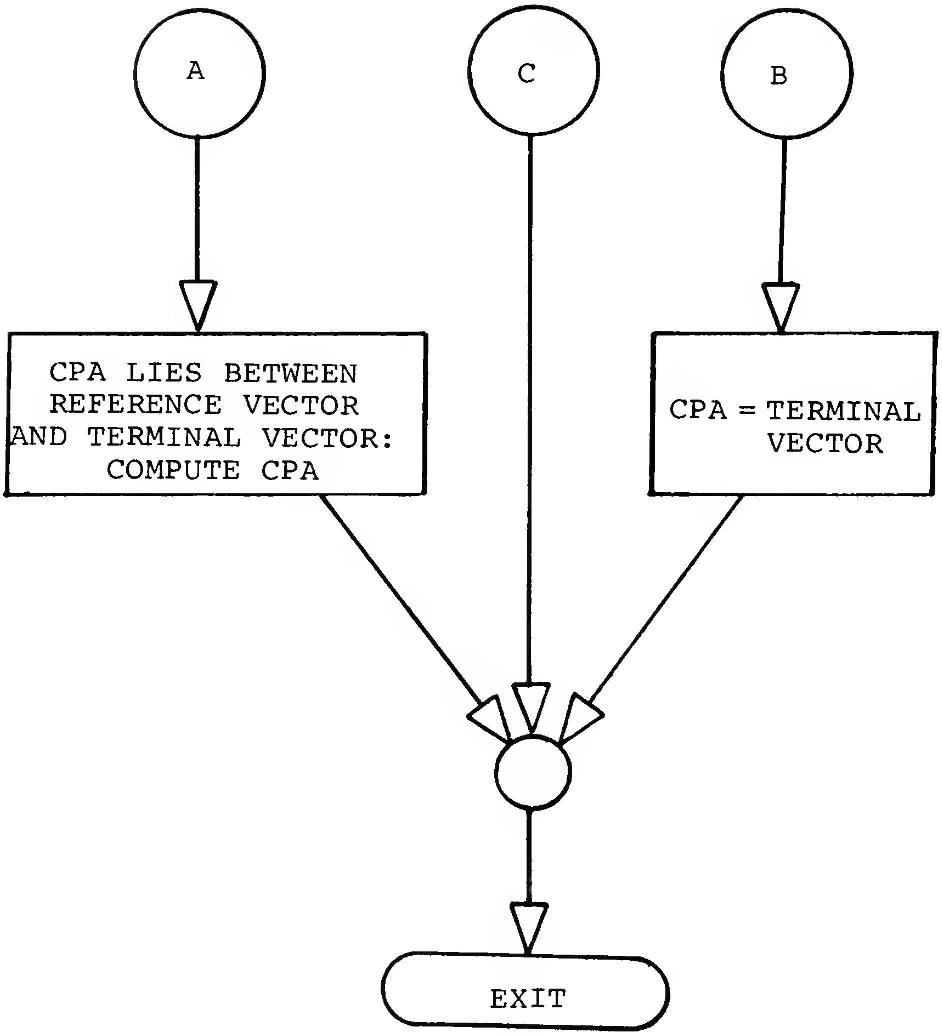


SUBROUTINE WYEB



SUBROUTINE CPACHK




A
C
B
CPA LIES BETWEEN REFERENCE VECTOR AND TERMINAL VECTOR: COMPUTE CPA
CPA = TERMINAL VECTOR
EXIT

APPENDIX C

```
C                    **************************
C                    * NUMERICAL INTEGRATION *
C                    *      BASIC MODEL       *
C                    **************************

      COMMON/ONE/SGN,YB,ZN
      INTEGER DALF

      DIMENSION UV(2),W1V(2),W2V(2),V1V(2),V2V(2),R1V(2),
     1RNV(2),UW1V(2),UW2V(2),TEMV(2),WV(2),VV(2),UWV(2)

      PI=3.1415926536

C           FORMAT CONTROL COUNTER (INITIALIZE)

      IZ=2
C           SUB SPEED
      V=10.

C           SAMPLE SIZE
      JKL=960

C           NO. OF ANGLE (ALFA) INCREMENTS
      IAI=9
      NA=5

C           NO. OF RHO INCREMENTS
      IBI=12

C           DEFINE BOTH SUB VECTORS
      CALL VINS(V1V,0.,V)
      CALL VINS(V2V,0.,-V)

C           SIZE OF DETECTION RADIUS (DEFINITE RANGE LAW)
      R=30.

C           LENGTH OF SUB PATROL
      PL=120.

C           INITIAL X-POSITION OF PATROL LINE
      XP=R

C           PRINT FIRST PAGE HEADING
      WRITE(6,600)R,PL,XP,V,JKL

C           INITIAL ANGLE
      ALF=0.

C           LOOP TO INCREMENT ALFA
      DO 800 IA=1,IAI

C           CONVERSION OF RADIANS TO DEGREES
      IF(IA-1)105,23,21
   23 DALF=ALF
      GO TO 22
   21 DALF=(IA+3)*5

C           INITIAL SHIP SPEED
   22 U=10.

C           CALCULATE POSITION OF BOTTOM OF PATROL LINE TO
C           INSURE CROSSING AT CENTER OF PATROL
      YP=XP*TAN(ALF)-(PL/2.)

C           LOOP TO INCREMENT RHO
      DO 801 IB=1,IBI
```

```
C          INITIALIZE LEG COUNT
      SLEG=0.

      COSA=COS(ALF)
      SINA=SIN(ALF)

C          DEFINE VECTOR OF SHIP VELOCITY
      CALL VINS(UV,U*COSA,U*SINA)

C          COMPUTE 1/V
      VI=1./V

C          COMPUTE SHIP/SUB SPEED RATIO
      RHO=U*VI

C          INITIALIZE SUB DIRECTION FOR DETERMINISTIC
C          CALCULATION
      ZN=1.

C          INITIALIZE SUB POSITION FOR DETERMINISTIC
C          CALCULATION
      YB=YP

C          INITIALIZE DETECTION COUNT
      DET=0.

C          COMPUTE MAXIMUM TIME FOR CALCULATIONS
      TMAX=(XP+R)/(U*COSA)+1.

C          DETERMINE RELATIVE SPEED VECTORS AND VALUES
C          OF MAGNITUDES AND ANGLES
      CALL VADD(V1V,UV,W1V,-1)
      CALL VADD(V2V,UV,W2V,-1)
      CALL VVAL(W1V,W1,UW1V,BW1,111)
      CALL VVAL(W2V,W2,UW2V,BW2,111)

C        DETERMINE IF DETECTION OCCURS

C          EACH COUNT OF THE LOOP INDEX REPRESENTS
C          ONE SAMPLE

      DO 8  J=1,JKL

C          DETERMINE SUB POSIT
C            INPUTS
C              XP:   X-DISTANCE TO PATROL LINE
C              YP:   Y-COORDINATE OF LOWER LIMIT OF PATROL
C                    LINE
C              PL:   LENGTH OF PATROL LINE

C            OUTPUTS
C              RC:   RANGE TO SUB'S INITIAL POSIT
C              BO:   BEARING OF SUB'S INITIAL POSIT
C              YB (VIA COMMON/ONE/):  INITIAL Y-COORDINATE
C                 OF THE SUBMARINE
C              R1V: INITIAL SUB POSITION VECTOR
      CALL WYEB(XP,YP,PL,RC,BC,R1V)

C          TIME TO END OF FIRST (RANDOM) LEG
      IF(SGN)700,105,702

C          TIME TO GO FOR SUB TO REACH LIMIT OF PATROL LINE
C          IF INITIAL SUB VELOCITY IS "DOWN"
  700 T1=(YB-YP)*VI
      GO TO 703

C          TIME TO GO FOR SUB TO REACH UPPER LIMIT OF PATROL
C          LINE
  702 T1=(YP+PL-YB)*VI
  703 T=T1
```

```
C           INITIALIZE TOTAL TIME
      TN=0.
C           INDICATE THAT THE FIRST (RANDOM) LEG IS TO BE EX-
C           AMINED
      MM=1
C           COMPUTE LENGTH OF RANDOM LEG
      GO TO 104
C           LOOP TO INCREMENT LEG COUNT
  103 DO 10 IN=1,MM
C           CHECK TO DETERMINE INITIAL DIRECTION OF SUB
      IF(SGN)101,105,102
C           INITIAL DIRECTION IS UP; USE PARAMETERS FOR THE
C           UP LEG
  101 TW=TW2
      CALL COPI(UWV,UW2V)
      W=W2
      CALL COPI(WV,W2V)
      CALL COPI(VV,V2V)
      GO TO 106
C           INITIAL DIRECTION IS DOWN; USE PARAMETERS FOR THE
C           DOWN LEG
  102 CALL COPI(UWV,UW1V)
      TW=TW1
      W=W1
      CALL COPI(WV,W1V)
      CALL COPI(VV,V1V)
C           REVERSE DIRECTION OF NEXT LEG
  106 SGN=-SGN
C           COMPUTE RELATIVE LENGTH OF THIS LEG
      CALL VMULT(UWV,TEMV,TW)
C           DETERMINE VECTOR FOR FINAL POSITION ON THIS LEG
      CALL VADD(R1V,TEMV,RNV,1)
      CALL CPACHK(R1V,RNV,UWV,RMIN,IN,J)
C           INCREMENT LEG COUNT
      SLEG=SLEG+1.
C           DETERMINE IF DETECTION OCCURRED ON THIS LEG
      IF(RMIN-R)3,3,11
C           NO DETECTION THIS LEG; DETERMINE IF SHIP IS OUT
C           OF RANGE BEYOND PATROL LINE
   11 TN=TN+T
    4 IF(TN-TMAX)6,8,8
C           IF THIS IS NOT THE FIRST (RANDOM) LEG, CONTINUE
    6 IF(MM-1)105,9,10
   10 CONTINUE
      GO TO 8
C           LENGTH OF TIME REQUIRED FOR SUB TO TRAVEL ENTIRE
C           SPAN OF PATROL LINE
    9 T=PL*VI
C           PROVIDE FOR POSSIBILITY OF 10000 LEGS MAXIMUM
      MM=10000
C           COMPUTE LENGTHS OF SECOND AND SUCCEEDING LEGS
  104 TW1=T*W1
      TW2=T*W2
      GO TO 103
```

```
C          INCREMENT CETECTICN CCUNTER
    3 DET=DET+1.
    8 CCNTINUE
      XJKL=JKL

C          COMPUTE PRCBABILITY OF DETECTION
      P=DET/XJKL
C          COMPUTE AVERAGE NO. CF LEGS FOR THIS SCENARIO
      XLEG=SLEG/XJKL

C          PRINT RESULTS FOR EACH SPEED RATIO
      WRITE(6,601)XLEG,U,RHC,W1,W2,DALF,P

C          INCREMENT SHIP SPEEC BY 5 KNCTS IF RHC<7,
C          THEN INCREMENT BY 10 KNOTS
      IF(IB-7)16,17,17
   16 U=U+5.
      GC TC 801
   17 U=U+10.
  801 CCNTINUE

C          FORMAT OUTPUT TO INCLUDE RESULTS FCR TWC
C          VALUES OF ALFA ON FIRST PAGE, AND THREE
C          VALUES OF ALFA CN SUBSEQUENT PAGES
      IF(IA-IZ)12,13,105

C          ONE LINE FEEC BETWEEN VALUES OF ALFA
   12 WRITE(6,602)
      GC TO 20

C          BEGIN NEW PAGE; INSERT CCLUMN HEADINGS
   13 IZ=IZ+3
      IF(IA-9)19,800,105
   19 WRITE(6,603)

C          INCREMENT ALFA BY 25 DEGREES CN THE FIRST
C          ITERATION, THEN BY 5 DEGREES ON SUBSEQUENT
C          ITERATIONS
   20 ALF=ALF+NA*(PI/36.)
  800 NA=1


  600 FCRMAT(1H1,15(/),T16,'PRCBABILITY OF RANDCM ENCOUNTER'
     1/T26,'BASIC MODEL'///T6,'DEFINITE RANGE LAW'/T8,'DETE'
     2,'CTION RANGE: ',F5.1,' MILES'//T6,'LENGTH OF SUB PAT'
     3,'ROL LINE: ',F5.1,' MILES'//T6,'X-DISTANCE TO PATROL'
     5,' LINE: ',F5.1,' MILES'//T6,'SUBMARINE SPEEC:',F5.1,
     5' KNOTS'//T6,'NC. OF SAMPLES TAKEN FOR EACH SCENARIO:'
     6,' ',I4///1X,'AVG.NO. SHIP  SPEED  RELATIVE SPEEDS    '
     7,'ROUTE  PROBABILITY OF'/2X,'LEGS   SPEED RATIO  UPLE'
     8,'G   DOWNLEG   ANGLE    DETECTION'//)
  601 FORMAT(1X,F5.2,1X,2(2X,F4.1),2(3X,F5.1),6X,I2,8X,F6.4)
  602 FCRMAT(/)
  603 FCRMAT(1H1,15(/),1X,'AVG.NO. SHIP  SPEEC  RELATIVE SP'
     1,'EEDS   ROUTE  PROBABILITY OF'/2X,'LEGS   SPEED RATI'
     2,'C  UPLEG   DCWNLEG   ANGLE    DETECTICN'//)

  105 STCP
      END
```

```
C                     **************************
C                     * NUMERICAL INTEGRATION *
C                     *    EXTENDED  MODEL    *
C                     **************************

      COMMON/ONE/SGN,YB,ZN

      DIMENSION UV(2),W1V(2),W2V(2),V1V(2),V2V(2),R1V(2),
     1RNV(2),UW1V(2),UW2V(2),TEMV(2),WV(2),VV(2),UWV(2)

      PI=3.1415926536

C          FORMAT CONTROL COUNTER (INITIALIZE)
      IZ=2

C          SUB SPEED
      V=10.

C          SAMPLE SIZE
      JKL=960

C          NO. OF TRACK CROSSING INCREMENTS
      IAI=10

C          NO. OF RHO INCREMENTS
      IBI=11

C          DEFINE BOTH SUB VECTORS
      CALL VINS(V1V,0.,V)
      CALL VINS(V2V,0.,-V)

C          SIZE OF DETECTION RADIUS (DEFINITE RANGE LAW)
      R=30.

C          LENGTH OF SUB PATROL
      PL=120.

C          INITIAL X-POSITION OF PATROL LINE
      XP=R

C          PRINT FIRST PAGE HEADING
      WRITE(6,600)R,PL,XP,V,JKL

C          INITIAL ANGLE
      ALF=0.
      YP=-PL/2.

C          LOOP TO INCREMENT CROSSING POSITION
      DO 800 IA=1,IAI

C          INITIAL SHIP SPEED
      U=10.

C          CALCULATE POSITION OF BOTTOM OF PATROL LINE TO
C          INSURE CROSSING AT CENTER OF PATROL

C          LOOP TO INCREMENT RHO
      DO 801 IB=1,IBI

C          DELETE DATA FOR 5<RHO<9
      IF(U-50.)20,20,21

C          GENERATE DATA FOR RHO=9
   21 IF(U-90.)17,22,105

C          INITIALIZE LEG COUNT
   20 SLEG=0.
```

```
   22 COSA=COS(ALF)
      SINA=SIN(ALF)
C           DEFINE VECTOR OF SHIP VELOCITY
      CALL VINS(UV,U*COSA,U*SINA)
C           COMPUTE 1/V
      VI=1./V
C           COMPUTE SHIP/SUB SPEED RATIO
      RHO=U*VI
C           INITIALIZE SUB DIRECTION FOR DETERMINISTIC
C           CALCULATION
      ZN=1.
C           INITIALIZE SUB POSITION FOR DETERMINISTIC
C           CALCULATION
      YB=YP
C           INITIALIZE DETECTION COUNT
      DET=0.
C           COMPUTE MAXIMUM TIME FOR CALCULATIONS
      TMAX=(XP+R)/(U*COSA)+1.
C           DETERMINE RELATIVE SPEED VECTORS AND VALUES
C           OF MAGNITUDES AND ANGLES
      CALL VADD(V1V,UV,W1V,-1)
      CALL VADD(V2V,UV,W2V,-1)
      CALL VVAL(W1V,W1,UW1V,BW1,111)
      CALL VVAL(W2V,W2,UW2V,BW2,111)
C         DETERMINE IF DETECTION OCCURS
C           EACH COUNT OF  INDEX J REPRESENTS ONE SAMPLE
      DO 8 J=1,JKL
C           DETERMINE SUB POSITION
C             INPUTS
C               XP:  X-DISTANCE TO PATROL LINE
C               YP:  Y-COORDINATE OF LOWER LIMIT OF PATROL
C                    LINE
C               PL:  LENGTH OF PATROL LINE
C             OUTPUTS
C               RO:  RANGE TO SUB'S INITIAL POSITION
C               BO:  BEARING OF SUB'S INITIAL POSITION
C               YB (VIA COMMON/ONE/):  INITIAL Y-COORDINATE
C                  OF SUBMARINE
C               R1V: INITIAL SUB POSITION VECTOR
      CALL WYEB(XP,YP,PL,RO,BO,R1V)
C           TIME TO END OF FIRST (RANDOM) LEG
      IF(SGN)700,105,702
C           TIME TO GO FOR SUB TO REACH LIMIT OF PATROL LINE
C           IF INITIAL SUB VELOCITY IS "DOWN"
  700 T1=(YB-YP)*VI
      GO TO 703
C           TIME TO GO FOR SUB TO REACH UPPER LIMIT OF PATROL
C           LINE
  702 T1=(YP+PL-YB)*VI
  703 T=T1
C           INITIALIZE TOTAL TIME
      TN=0.
C           INDICATE THAT THE FIRST (RANDOM) LEG IS TO BE EX-
C           AMINED
      MM=1
```

```
C           COMPUTE LENGTH OF RANDOM LEG
      GO TO 104
C           LOOP TO INCREMENT LEG COUNT
  103 DO 10 IN=1,MM
C           CHECK TO DETERMINE INITIAL DIRECTION OF SUB
      IF(SGN)101,105,102
C           INITIAL DIRECTION IS UP; USE PARAMETERS FOR THE
C           UP LEG
  101 TW=TW2
      CALL COPI(UWV,UW2V)
      W=W2
      CALL COPI(WV,W2V)
      CALL COPI(VV,V2V)
      GO TO 106
C           INITIAL DIRECTION IS DOWN; USE PARAMETERS FOR THE
C           DOWN LEG
  102 CALL COPI(UWV,UW1V)
      TW=TW1
      W=W1
      CALL COPI(WV,W1V)
      CALL COPI(VV,V1V)
C           REVERSE DIRECTION OF NEXT LEG
  106 SGN=-SGN
C           COMPUTE RELATIVE LENGTH OF THIS LEG
      CALL VMULT(UWV,TEMV,TW)
C           DETERMINE VECTOR FOR FINAL POSITION ON THIS LEG
      CALL VADD(R1V,TEMV,RNV,1)
      CALL CPACHK(R1V,RNV,UWV,RMIN,IN,J)
C           INCREMENT LEG COUNT
      SLEG=SLEG+1.
C           DETERMINE IF DETECTION OCCURRED ON THIS LEG
      IF(RMIN-R)3,3,11
C           NO DETECTION THIS LEG; DETERMINE IF SHIP IS OUT
C           OF RANGE BEYOND PATROL LINE
   11 TN=TN+T
    4 IF(TN-TMAX)6,8,8
C           IF THIS IS NOT THE FIRST (RANDOM) LEG, CONTINUE
    6 IF(MM-1)105,9,10
   10 CONTINUE
      GO TO 8
C           LENGTH OF TIME REQUIRED FOR SUB TO TRAVEL ENTIRE
C           SPAN OF PATROL LINE
    9 T=PL*VI
C           PROVIDE FOR POSSIBILITY OF 10000 LEGS MAXIMUM
      MM=10000
C           COMPUTE LENGTHS OF SECOND AND SUCCEEDING LEGS
  104 TW1=T*W1
      TW2=T*W2
      GO TO 103
C           INCREMENT DETECTION COUNTER
    3 DET=DET+1.
    8 CONTINUE
      XJKL=JKL
```

```
C           COMPUTE PROBABILITY OF DETECTION
      P=DET/XJKL
C           COMPUTE AVERAGE NO. OF LEGS FOR THIS SCENARIO
      XLEG=SLEG/XJKL
C           GENERATE CROSSING COORDINATE FOR PRINTOUT
      XYP=-YP
C           PRINT RESULTS FOR EACH SPEED RATIO
      WRITE(6,601)XLEG,U,RHC,W1,W2,XYP,P
C           INCREMENT SHIP SPEED BY 5 KNOTS IF RHC<3,
C           THEN BY 10 KNOTS
      IF(IB-5)16,17,17
   16 U=U+5.
      GO TO 801
   17 U=U+10.
  801 CONTINUE
C           FORMAT OUTPUT TO INCLUDE RESULTS FOR TWO
C           VALUES OF ALFA ON EACH PAGE
      IF(IA-IZ)12,13,105
C           ONE LINE FEED BETWEEN VALUES OF ALFA
   12 WRITE(6,602)
      GO TO 18
C           BEGIN NEW PAGE; INSERT COLUMN HEADINGS
   13 IZ=IZ+2
      IF(IA-10)19,18,105
   19 WRITE(6,603)
C           INCREMENT CROSSING POSITION BY 5 MILES
C           UNTIL WITHIN 30 MILES OF PATROL TERMINUS,
C           THEN INCREMENT BY 20 MILES
   18 IF(IA-7)14,15,15
   14 YP=YP-5.
      GO TO 800
   15 YP=YP-20.
  800 CONTINUE

  600 FORMAT(1H1, 8(/),T16,'PROBABILITY OF RANDOM ENCOUNTER'
     1/T24,'EXTENDED MODEL'
     1///T6,'DEFINITE RANGE LAW'/T8,'DETECTION RANGE: ',F5.1
     2,' MILES'//T6,'LENGTH OF SUB PATROL LINE: ',F5.1,' MI'
     3,'LES'//T6,'X-DISTANCE TO PATROL LINE: ',F5.1,' MILES'
     4//T6,'CROSSING ANGLE: 0.0 DEGREES (PERPENDICULAR)'
     5//T6,'SUBMARINE SPEED:',F5.1,' KNOTS'//T6,'NO. OF SAM'
     6,'PLES TAKEN FOR EACH SCENARIO: ',I4///1X,'AVG.NO. SH'
     7,'IP  SPEED  RELATIVE SPEEDS   CROSS  PROBABILITY OF'/
     82X,'LEGS    SPEED RATIO  UPLEG    DOWNLEG    POSIT      DE'
     9,'TECTION'//)
  601 FORMAT(1X,F5.2,1X,2(2X,F4.1),2(3X,F5.1),5X,F5.1,7X,
     1F6.4)
  602 FORMAT(/)
  603 FORMAT(1H1,17(/),1X,'AVG.NO. SHIP  SPEED  RELATIVE SP'
     1,'EEDS   CROSS  PROBABILITY OF'/2X,'LEGS    SPEED RATI'
     2,'O  UPLEG    DOWNLEG    POSIT     DETECTION'//)

  105 STOP
      END
```

```
C                 ********************************
C                 * NUMERICAL INTEGRATION MODELS *
C                 *          SUBROUTINES         *
C                 ********************************


      SUBROUTINE WYEB(XP,YP,PL,RC,BC,A)
C
C          THIS SUBROUTINE GENERATES SEQUENTIAL Y-
C          COORDINATES ON A SPECIFIED LINE SEGMENT;
C          WHEN AN END POINT OF THE LINE IS GENERATED,
C          THE DIRECTION OF THE SEQUENCE IS REVERSED
C
C          INPUTS
C            YB=Y-POSITION OF SUB ON PATROL LINE
C            SGN=INITIAL DIRECTION OF SUB
C
      COMMON/ONE/SGN,YB,ZN
      DIMENSION A(2),B(2)
C
C          INCREMENT SUB POSITION BY 500 YDS IF ZN=1;
C          DECREMENT SUB POSIT BY 500 YDS IF ZN=-1
      YB=YB+0.25*ZN
C
C          REVERSE SUB COURSE WHEN IT REACHES END OF
C          PATROL LINE
      IF(ZN*(2.*(YP-YB)+PL)+PL)9,9,10
C
C          POSITION SUB EXACTLY AT TERMINUS OF PATROL
C          LINE PRIOR TO TURN
    9 YB=YP+0.5*PL*(1.+ZN)
C
C          REVERSE COURSE OF SUB
      ZN=-ZN
   10 SGN=ZN
C
C          DETERMINE INITIAL RANGE AND BEARING TO SUB
      A(1)=XP
      A(2)=YB
      CALL VVAL(A,RO,B,BC,101)
      RETURN
      END
```

```
      SUBROUTINE VINS(A,X,Y)

C          THIS SUBROUTINE INSERTS THE X AND Y VECTOR
C          COMPONENTS INTO ARRAY A

      DIMENSION A(1)
      A(1)=X
      A(2)=Y
      RETURN
      END




      SUBROUTINE COPI(A,B)

C          THIS SUBROUTINE GENERATES THE VECTOR A SUCH THAT
C          FOR A GIVEN VECTOR B, A=B

      DIMENSION A(1),B(1)
      DO 1 I=1,2
    1 A(I)=B(I)
      RETURN
      END




      SUBROUTINE VADD(A,B,R,N)

C        THIS SUBROUTINE PERFORMS THE FOLLOWING VECTOR
C        OPERATIONS:
C          IF N>0: R=A+B (VECTOR ADDITION)
C          IF N<0: R=A-B (VECTOR SUBTRACTION)
C          IF N=0: THE DOT PRODUCT OF A AND B IS FORMED AND
C           RESULT IS PLACED IN THE FIRST MEMBER OF ARRAY R

      DIMENSION A(1),B(1),R(1)
      IF(N)1,2,3
C          ADD VECTORS
    3 DO 4 I=1,2
    4 R(I)=A(I)+B(I)
      RETURN
C          SUBTRACT VECTORS
    1 DO 5 I=1,2
    5 R(I)=A(I)-B(I)
      RETURN
C          COMPUTE DOT PRODUCT OF VECTORS
    2 DO 6 I=1,2
    6 R(I)=A(I)*B(I)
      R(1)=R(1)+R(2)
      RETURN
      END
```

```
      SUBROUTINE VMULT(A,B,X)

C           THIS SUBROUTINE MULTIPLIES THE VECTOR A BY THE
C           SCALAR X TC FORM THE VECTOR B

      DIMENSION A(1),B(1)
      DO 1 I=1,2
    1 B(I)=X*A(I)
      RETURN
      END




      SUBROUTINE VVAL(A,VL,UV,ANGLE,M)

C           THIS SUBROUTINE DETERMINES THE FOLLOWING
C           VALUES OF A:
C           A=INPUT VECTOR (SUPPLIED BY THE CALLING PROGRAM)
C           VL=MAGNITUDE OF A
C           UV=UNIT VECTOR PARALLEL TO A
C           ANGLE=ANGLE OF A MEASURED CCW FROM X-AXIS

      DIMENSION A(1),UV(1)
      IF(M-100)2,3,4
    4 IF(M-110)5,6,7

C           COMPUTE VECTOR LENGTH ONLY
    3 VL=SQRT(A(1)**2+A(2)**2)
      RETURN

C           COMPUTE ANGLE OF VECTOR ONLY
    2 ANGLE=ATAN2(A(2),A(1))
      RETURN

C           COMPUTE BOTH LENGTH AND ANGLE OF VECTOR
    5 VL=SQRT(A(1)**2+A(2)**2)
      GO TO 2

C           GENERATE UNIT VECTOR ONLY
    6 VL=SQRT(A(1)**2+A(2)**2)
      DO 1 I=1,2
    1 UV(I)=A(I)/VL
      RETURN

C           COMPUTE ANGLE AND GENERATE UNIT VECTOR
    7 ANGLE=ATAN2(A(2),A(1))
      GO TO 6
      END
```

```
      SUBROUTINE CPACHK(R1,RN,UWV,RCPA,IN,J)

C             THIS SUBROUTINE DETERMINES THE CLOSEST POINT
C             TO THE ORIGIN OF ANY LINE SEGMENT, THE END
C             POINTS OF WHICH ARE DEFINED BY VECTORS

C           R1=REFERENCE VECTOR
C           RN=TERMINAL VECTOR
C           UWV=UNIT VECTOR PARALLEL TO DRM
C           RCPA=DISTANCE AT CLOSEST POINT OF APPROACH (CPA)
C           IN=LEG NO. BEING EXAMINED AT THIS EXECUTION
C           J=SAMPLE NO. BEING EXAMINED AT THIS EXECUTION


      DIMENSION R1(1),RN(1),UWV(1),X(2),Y(2),CPA(2),Z(2),
     1RT(2)
      COMMON/TWC/Y,N

C             COMPUTE DOT PRODUCT: R1*UWV=X
      CALL VADD(R1,UWV,X,0)
      IF(X(1))1,2,2

C             CPA WILL OCCUR ON THIS LEG; COMPUTE THE DOT PROD-
C             UCT: RN*UWV=X
    1 CALL VADD(RN,UWV,X,0)
      IF(X(1))3,3,4

C             CPA WILL NOT OCCUR ON THIS LEG; CPA=R1
    2 CALL VVAL(R1,RCPA,X,D,100)
      RETURN

C             CPA WAS NOT REACHED ON THIS LEG;  CPA=RN
    3 CALL VVAL(RN,RCPA,X,D,100)
      RETURN

C             DETERMINE CPA; ASSUME LEG HAS LENGTH < 250 MILES
    4 DO 5 I=1,500

C             USE 2-MILE INCREMENTS
      P=2.*I

C             INCREMENT DRM BY 2 MILES
      CALL VMULT(UWV,Y,P)

C             ADD TO REFERENCE VECTOR
      CALL VADD(R1,Y,X,1)

C             COMPUTE DOT PRODUCT  X*UWV=Z
      CALL VADD(X,UWV,Z,0)

C             IF Z BECOMES POSITIVE, MAKE FINE INCREMENT CHECK
C             BELOW; IF Z(1)=0, X=CPA; OTHERWISE TRY AGAIN
      IF(Z(1))5,6,7

C             INCREMENT DRM BY 0.1 MILE

C             PRESERVE PRECEDING VECTOR
    7 CALL VMULT(UWV,Y,P-2.)
      CALL VADD(R1,Y,X,1)

C             CHECK LAST 2 MILES OF DRM IN 0.1-MILE INCREMENTS
      DO 8 I=1,21
      P=0.1*I

C             INCREMENT DRM BY 0.1 MILE AND FIND CPA
      CALL VMULT(UWV,Y,P)
      CALL VADD(X,Y,RT,1)
      CALL VADD(RT,UWV,Z,0)
```

```
      IF(Z(1))8,6,6
    8 CONTINUE
C           ERROR EXISTS IN PROGRAM IF NEXT INSTRUCTION IS
C           EXECUTED
      WRITE(6,61)IN,J
   61 FORMAT(//10X,'LEG',1X,I4,1X,'OF SAMPLE',I5,1X,'GENER',
     1'ATED ERROR IN CPACHK ROUTINE'//)
      IF(Z(1)) 5,6,7
    5 CONTINUE
C           DRM LEG>250 MILES IF NEXT INSTRUCTION IS
C           EXECUTED
      WRITE(6,60)IN,J
   60 FORMAT(//10X,'LEG',1X,I4,1X,'OF SAMPLE',I5,1X,'IS
     1GREATER THAN 250 MILES'//)
      RETURN
C           COMPUTE CPA DISTANCE
    6 CALL VVAL(X,RCPA,Y,D,100)
      RETURN
      END
```

APPENDIX D

```
C                   ***********************
C                   *   RANDOM SAMPLING   *
C                   *  MIDPOINT CROSSING  *
C                   ***********************

      COMMON/ONE/SGN,YB

      DIMENSION UV(2),W1V(2),W2V(2),V1V(2),V2V(2),R1V(2),
     1ENV(2),UW1V(2),UW2V(2),TEMV(2),WV(2),VV(2),UWV(2)

C        INITIALIZE URN GENERATOR

      POSIT=URN(0)

      PI=3.1415926536

C        SUB SPEED

      V=10.

C        SAMPLE SIZE

      JKL=1000

C        NO. OF ANGLE (ALFA) INCREMENTS

      IAI=17

C        NO. OF RHO INCREMENTS

      IRI=9

C        DEFINE BOTH SUB VECTORS

      CALL VINS(V1V,0.,V)
      CALL VINS(V2V,0.,-V)

C        SIZE OF DETECTION RADIUS (DEFINITE RANGE LAW)

      R=30.

C        LENGTH OF SUB PATROL

      PL=120.

C        INITIAL X-POSITION OF PATROL LINE

      XP=R

      WRITE(6,600)R,PL,XP,V,JKL

C        INITIAL ANGLE

      ALF=0.0

C        LOOP TO INCREMENT ALFA

      DO 800 IA=1,IAI

C        INITIAL SHIP SPEED

      U=10.

C        CALCULATE POSITION OF BOTTOM OF PATROL LINE TO
C        INSURE CROSSING AT CENTER OF PATROL

      YP=XP*TAN(ALF)-(PL/2.)
```

```
C         LOOP TO INCREMENT RHO
      DO 801 IB=1,IBI
C         INITIALIZE LEG COUNT
      SLEG=0.
      COSA=COS(ALF)
      SINA=SIN(ALF)
C         DEFINE VECTOR OF SHIP VELOCITY
      CALL VINS(UV,U*COSA,U*SINA)
C         COMPUTE SHIP/SUB SPEED RATIO
      RHO=U/V
C         INITIALIZE SUB DIRECTION FOR DETERMINISTIC
C         CALCULATION
      SGN=1.
C         INITIALIZE DETECTION COUNT
      DET=0.
C         COMPUTE MAXIMUM TIME FOR CALCULATIONS
      TMAX=(XP+R)/(U*COSA)+1.
C         DETERMINE RELATIVE SPEED VECTORS AND VALUES OF
C         MAGNITUDES AND ANGLES
      CALL VADD(V1V,UV,W1V,-1)
      CALL VADD(V2V,UV,W2V,-1)
      CALL VVAL(W1V,W1,UW1V,BW1,111)
      CALL VVAL(W2V,W2,UW2V,BW2,111)
C         DETERMINE IF DETECTION OCCURS
C         EACH COUNT OF THE LOOP INDEX REPRESENTS ONE SAMPLE
      DO 8 J=1,JKL
C         DETERMINE SUB POSIT (UNIFORMLY DISTRIBUTED)
C             INPUTS
C               XP:  X-DISTANCE TO PATROL LINE
C               YP:  Y-COORDINATE OF LOWER LIMIT OF PATROL
C                    LINE
C               PL:  LENGTH OF PATROL LINE
C            OUTPUTS
C               RO:  RANGE TO SUB'S INITIAL POSIT
C               BO:  BEARING OF SUB'S INITIAL POSIT
C               YB (VIA COMMON/ONE/):  INITIAL Y-COORDINATE
C                  SUB
C               R1V: INITIAL SUB POSITION VECTOR
      CALL WYEB(XP,YP,PL,RO,BO,R1V)
C         TIME TO END OF FIRST (RANDOM) LEG
      IF(SGN)700,105,702
C         TIME TO GO FOR SUB TO REACH LIMIT OF PATROL LINE
C         IF INITIAL SUB VELOCITY IS "DOWN"
  700 T1=(YB-YP)/V
      GO TO 703
C         TIME TO GO FOR SUB TO REACH UPPER LIMIT OF PATROL
```

```
C          LINE
  202 T1=(YP+PL-YB)/V
  203 T=T1
C          INITIALIZE TOTAL TIME
      TN=0.
C          INDICATE THAT THE FIRST (RANDOM) LEG IS TO BE
C          EXAMINED
      MM=1
C          COMPUTE LENGTH OF RANDOM LEG
      GO TO 104
C          LOOP TO INCREMENT LEG COUNT
  103 DO 10 IN=1,MM
C          CHECK TO DETERMINE INITIAL DIRECTION OF SUB
      IF(SGN)101,105,102
C          INITIAL DIRECTION IS UP; USE PARAMETERS FOR THE
C          UP LEG
  101 TW=TW2
      CALL COPI(UWV,UW2V)
      W=W2
      CALL COPI(WV,W2V)
      CALL COPI(VV,V2V)
      GO TO 106
C          INITIAL DIRECTION IS DOWN; USE PARAMETERS FOR THE
C          DOWN LEG
  102 CALL COPI(UWV,UW1V)
      TW=TW1
      W=W1
      CALL COPI(WV,W1V)
      CALL COPI(VV,V1V)
C          REVERSE DIRECTION OF NEXT LEG
  106 SGN=-SGN
C          COMPUTE RELATIVE LENGTH OF THIS LEG
      CALL VMULT(UWV,TEMV,TW)
C          DETERMINE VECTOR FOR FINAL POSITION ON THIS LEG
      CALL VADD(R1V,TEMV,RNV,1)
      CALL CPACHK(R1V,RNV,UWV,RMIN,IN,J)
C          INCREMENT LEG COUNT
      SLEG=SLEG+1.
C          DETERMINE IF DETECTION OCCURRED ON THIS LEG
      IF(RMIN-R)3,3,11
   11 TN=TN+T
    4 IF(TN-TMAX)6,8,8
    6 IF(MM-1)105,9,10
   10 CONTINUE
      GO TO 8
    9 T=PL/V
      MM=10000
```

```
104 TW1=T*W1
    TW2=T*W2
    GO TO 103
  3 DET=DET+1.
  8 CONTINUE
    XJKL=JKL
    P=DET/XJKL
    XLEG=SLEG/XJKL
    WRITE(6,601)XLEG,U,RHO,W1,W2,ALF,P
801 U=U+10.
    WRITE(6,602)
602 FORMAT(/)
800 ALF=ALF+PI/36.
600 FORMAT(1H1////T16,'PROBABILITY OF RANDOM ENCOUNTER',/
   1//T6,'DEFINITE RANGE LAW'/T8,'DETECTION RANGE: ',F5.1,
   2' MILES'//T6,'LENGTH OF SUB PATROL LINE: ',F5.1,' MIL'
   3,'ES'//T6,'X-DISTANCE TO PATROL LINE:',F5.1,' MILES'//
   4T6,'SUBMARINE SPEED:',F5.1,' KNOTS'//T6,'NO. OF SAMPL'
   5,'ES TAKEN FOR EACH SCENARIO: ',I4///T3,' AVG. NO. SH'
   6,'IP  SPEED '
   7,'RELATIVE SPEED  ALFA   PROB. DETECTION'/T5,'LEGS    '
   8,' SPEED RATIO UPLEG  DOWNLEG'//)
601 FORMAT(T4,F5.2,2X,3(1X,F5.1),2X,F5.1,3X,F6.4,7X,F6.4)
105 STOP
    END
```

```
      SUBROUTINE WYEB(XP,YP,PL,RO,BO,A)
C          INPUTS
C            YB=Y-POSITION OF SUB ON PATROL LINE
C            YB IS UNIFORMLY DISTRIBUTED
C            SGN=INITIAL DIRECTION (EQUALLY LIKELY) OF SUB
C            VELOCITY
      COMMON/ONE/SGN,YB
      DIMENSION A(2),B(2)
C          DETERMINE SUB POSIT RANDOMLY
      YB=YP+URN(1)*PL
C          DETERMINE INITIAL DIRECTION
    4 IF(URN(1)-0.5)5,6,7
    6 GO TO 4
C          UP
    5 SGN=1.
      GO TO 8
C          DOWN
    7 SGN=-1.
C          DETERMINE INITIAL RANGE AND BEARING TO SUB
    8 A(1)=XP
      A(2)=YB
      CALL VVAL(A,RO,B,BO,101)
      RETURN
      END
```

```
      SUBROUTINE CPACHK(R1,RN,UWV,RCPA,IN,J)
      DIMENSION R1(1),RN(1),UWV(1),X(2),Y(2),CPA(2),Z(2)
      COMMON/TWO/Y,N
C        R1=REFERENCE VECTOR
C        RN=TERMINAL VECTOR
C        UWV=UNIT VECTOR OF DRM

C        COMPUTE DOT PRODUCT: R1*UWV=X
      CALL VADD(R1,UWV,X,0)
      IF(X(1))1,2,2
C        CPA WILL OCCUR ON THIS LEG: COMPUTE THE DOT
C        PRODUCT: RN*UWV=X
    1 CALL VADD(RN,UWV,X,0)
      IF(X(1))3,3,4
C        CPA WILL NOT OCCUR ON THIS LEG: CPA=R1  COMPUTE
C        CPA
    2 CALL VVAL(R1,RCPA,X,D,100)
      RETURN
C        CPA WAS NOT REACHED ON THIS LEG:  CPA=RN: COMPUTE
C        CPA
    3 CALL VVAL(RN,RCPA,X,D,100)
      RETURN
C        DETERMINE CPA: ASSUME LEG HAS LENGTH< 250 MILES
    4 DO 5 I=1,500
C        USE HALF-MILE INCREMENTS
      P=I/2.
C        INCREMENT DRM BY 1/2 MILE
      CALL VMULT(UWV,Y,P)
C        ADD TO REFERENCE VECTOR
      CALL VADD(R1,Y,X,1)
C        COMPUTE DOT PRODUCT   X DOT UWV=Z
      CALL VADD(X,UWV,Z,0)
C        IF Z CHANGES SIGN, OR BECOMES ZERO VECTOR X=CPA:
C        OTHERWISE TRY AGAIN
      IF(Z(1)) 5,6,6
    5 CONTINUE
C        DRM LEG>250 MILES
      WRITE(6,60)IN,J
   60 FORMAT(//10X,'LEG',1X,I4,1X,'OF SAMPLE',I5,1X,'IS
     1GREATER THAN 250 MILES'//)
      RETURN
    6 CALL VVAL(X,RCPA,Y,D,100)
      RETURN
      END
```

```
      SUBROUTINE VINS(A,X,Y)
C         THIS SUBROUTINE INSERTS THE X AND Y VECTOR
C         COMPONENTS INTO ARRAY A
      DIMENSION A(1)
      A(1)=X
      A(2)=Y
      RETURN
      END
```

```
      SUBROUTINE COPI(A,B)
C         THIS SUBROUTINE GENERATES THE VECTOR A SUCH THAT
C         FOR A GIVEN VECTOR B, A=B
      DIMENSION A(1),B(1)
      DO 1 I=1,2
    1 A(I)=B(I)
      RETURN
      END
```

```
      SUBROUTINE VADD(A,B,R,N)
C         THIS SUBROUTINE DOES THE FOLLOWING VECTOR
C         OPERATIONS:
C            IF N>0: R=A+B
C            IF N<0: R=A-B
C            IF N=0: THE DOT PRODUCT OF A AND B IS FORMED AND
C         RESULT IS PLACED IN THE FIRST MEMBER OF ARRAY R
      DIMENSION A(1),B(1),R(1)
      IF(N)1,2,3
C         ADD VECTORS
    3 DO 4 I=1,2
    4 R(I)=A(I)+B(I)
      RETURN
C         SUBTRACT VECTORS
    1 DO 5 I=1,2
    5 R(I)=A(I)-B(I)
      RETURN
C         COMPUTE DOT PRODUCT OF VECTORS
    2 DO 6 I=1,2
    6 R(I)=A(I)*B(I)
      R(1)=R(1)+R(2)
      RETURN
      END
```

```
      SUBROUTINE VMULT(A,B,X)
C          THIS SUBROUTINE MULTIPLIES THE VECTOR A BY THE
C          SCALAR X TO FORM THE VECTOR B
      DIMENSION A(1),B(1)
      DO 1 I=1,2
    1 B(I)=X*A(I)
      RETURN
      END
```

```
      SUBROUTINE VVAL(A,VL,UV,ANGLE,M)
C          THIS SUBROUTINE DETERMINES THE FOLLOWING
C          VALUES OF A:
C             A=INPUT VECTOR (SUPPLIED BY THE CALLING PROGRAM)
C             VL=MAGNITUDE OF A
C             UV=UNIT VECTOR PARALLEL TO A
C             ANGLE=ANGLE OF A MEASURED CCW FROM X-AXIS
      DIMENSION A(1),UV(1)
      IF(M-100)2,3,4
    4 IF(M-110)5,6,7
C          COMPUTE VECTOR LENGTH ONLY
    3 VL=SQRT(A(1)**2+A(2)**2)
      RETURN
C          COMPUTE ANGLE OF VECTOR ONLY
    2 ANGLE=ATAN2(A(2),A(1))
      RETURN
C          COMPUTE BOTH LENGTH AND ANGLE OF VECTOR
    5 VL=SQRT(A(1)**2+A(2)**2)
      GO TO 2
C          GENERATE UNIT VECTOR ONLY
    6 VL=SQRT(A(1)**2+A(2)**2)
      DO 1 I=1,2
    1 UV(I)=A(I)/VL
      RETURN
C          COMPUTE ANGLE AND GENERATE UNIT VECTOR
    7 ANGLE=ATAN2(A(2),A(1))
      GO TO 6
      END
```

```
C                 ****************************
C                 *       RANDOM SAMPLING     *
C                 *  NORMALIZED CONVOY ROUTE  *
C                 ****************************

      COMMON/ONE/SGN,YB
      DIMENSION UV(2),W1V(2),W2V(2),V1V(2),V2V(2),R1V(2),
     1RNV(2),UW1V(2),UW2V(2),TEMV(2),WV(2),VV(2),UWV(2)
      REAL NORM
C        INITIALIZE URN GENERATOR
      POSIT=URN(0)
      PI=3.1415926536
C        SUB SPEED
      V=10.
C        SAMPLE SIZE
      JKL=1000
C        NO. OF ANGLE (ALFA) INCREMENTS
      IAI=17
C        NO. OF RHO INCREMENTS
      IBI=9
C        DEFINE BOTH SUB VECTORS
      CALL VINS(V1V,0.,V)
      CALL VINS(V2V,0.,-V)
C        SIZE OF DETECTION RADIUS (DEFINITE RANGE LAW)
      R=30.
C        LENGTH OF SUB PATROL
      PL=120.
C        INITIAL X-POSITION OF PATROL LINE
      XP=R
      WRITE(6,600)R,PL,XP,V,JKL
C        INITIAL ANGLE
      ALF=0.0
C        LOOP TO INCREMENT ALFA
      DO 800 IA=1,IAI
C        INITIAL SHIP SPEED
      U=10.
C        LOOP TO INCREMENT RHO
```

```
      DO 801 IP=1,IBI
C        INITIALIZE LEG COUNT
      SLEG=0.
      COSA=COS(ALF)
      SINA=SIN(ALF)
C        DEFINE VECTOR OF SHIP VELOCITY
      CALL VINS(UV,U*COSA,U*SINA)
C        COMPUTE SHIP/SUB SPEED RATIO
      RHO=U/V
C        INITIALIZE SUB DIRECTION FOR DETERMINISTIC
C        CALCULATION
      SGN=1.
C        INITIALIZE DETECTION COUNT
      DET=0.
C        COMPUTE MAXIMUM TIME FOR CALCULATIONS
      TMAX=(XP+R)/(U*COSA)+1.
C        DETERMINE RELATIVE SPEED VECTORS AND VALUES OF
C        MAGNITUDES AND ANGLES
      CALL VADD(V1V,UV,W1V,-1)
      CALL VADD(V2V,UV,W2V,-1)
      CALL VVAL(W1V,W1,UW1V,BW1,111)
      CALL VVAL(W2V,W2,UW2V,BW2,111)
C        NORMALIZE CONVOY TRACK FOR EACH UNIFORMLY
C        DISTRIBUTED SUBMARINE STARTING POSITION
C        SAMPLE SIZE FOR NORMAL IS DETERMINED BY THIS LOOP
      DO 7 JJ=1,1000
      YP=NORM(20.)-(PL/2.)
C        DETERMINE IF DETECTION OCCURS
C        EACH COUNT OF THE LOOP INDEX REPRESENTS ONE SAMPLE
      DO 8 J=1,JKL
C        DETERMINE SUB POSIT (UNIFORMLY DISTRIBUTED)
C              INPUTS
C                XP:  X-DISTANCE TO PATROL LINE
C                YP:  Y-COORDINATE OF LOWER LIMIT OF PATROL
C                     LINE
C                PL:  LENGTH OF PATROL LINE
C
C              OUTPUTS
C                RO:  RANGE TO SUB'S INITIAL POSIT
C                BO:  BEARING OF SUB'S INITIAL POSIT
C                YB (VIA COMMON/ONE/):  INITIAL Y-COORDINATE
C                     SUB
C                R1V: INITIAL SUB POSITION VECTOR
      CALL WYER(XP,YP,PL,RO,BO,R1V)
C        TIME TO END OF FIRST (RANDOM) LEG
      IF(SGN)700,105,702
C        TIME TO GO FOR SUB TO REACH LIMIT OF PATROL LINE
C        IF INITIAL SUB VELOCITY IS "DOWN"
```

```
  701 T1=(YB-YP)/V
      GO TO 703
C         TIME TO GO FOR SUB TO REACH UPPER LIMIT OF PATROL
C         LINE
  702 T1=(YP+PL-YB)/V
  703 T=T1
C         INITIALIZE TOTAL TIME
      TN=0.
C         INDICATE THAT THE FIRST (RANDOM) LEG IS TO BE
C         EXAMINED
      MM=1
C         COMPUTE LENGTH OF RANDOM LEG
      GO TO 104
C         LOOP TO INCREMENT LEG COUNT
  103 DO 10 IN=1,MM
C         CHECK TO DETERMINE INITIAL DIRECTION OF SUB
      IF(SGN)101,105,102
C         INITIAL DIRECTION IS UP: USE PARAMETERS FOR THE
C         UP LEG
  101 TW=TW2
      CALL COPI(UWV,UW2V)
      W=W2
      CALL COPI(WV,W2V)
      CALL COPI(VV,V2V)
      GO TO 106
C         INITIAL DIRECTION IS DOWN: USE PARAMETERS FOR THE
C         DOWN LEG
  102 CALL COPI(UWV,UW1V)
      TW=TW1
      W=W1
      CALL COPI(WV,W1V)
      CALL COPI(VV,V1V)
C         REVERSE DIRECTION OF NEXT LEG
  106 SGN=-SGN
C         COMPUTE RELATIVE LENGTH OF THIS LEG
      CALL VMULT(UWV,TEMV,TW)
C         DETERMINE VECTOR FOR FINAL POSITION ON THIS LEG
      CALL VADD(R1V,TEMV,RNV,1)
      CALL CPACHK(R1V,RNV,UWV,RMIN,IN,J)
C         INCREMENT LEG COUNT
      SLEG=SLEG+1.
C         DETERMINE IF DETECTION OCCURRED ON THIS LEG
      IF(RMIN-R)3,3,11
   11 TN=TN+T
    4 IF(TN-TMAX)6,8,8
```

```
    5 IF(MM-1)105,9,10
   10 CONTINUE
      GO TO 8
    9 T=PL/V
      MM=10000
  104 TW1=T*W1
      TW2=T*W2
      GO TO 103
    3 DET=DET+1.
    8 CONTINUE
    7 CONTINUE
      XJKL=JKL*1000
      P=DET/XJKL
      XLEG=SLEG/XJKL
      WRITE(6,601)XLEG,U,RHO,W1,W2,ALF,P
  301 U=U+10.
      WRITE(6,602)
  602 FORMAT(/)
  300 ALF=ALF+PI/36.
  600 FORMAT(1H1////T16,'PROBABILITY OF RANDOM ENCOUNTER',/
     1//T6,'DEFINITE RANGE LAW'/T8,'DETECTION RANGE: ',F5.1,
     2' MILES'//T6,'LENGTH OF SUB PATROL LINE: ',F5.1,' MIL'
     3,'ES'//T6,'X-DISTANCE TO PATROL LINE:',F5.1,' MILES'//
     4T6,'SUBMARINE SPEED:',F5.1,' KNOTS'//T6,'NO. OF SAMPL'
     5,'ES TAKEN FOR EACH SCENARIO: ',I4///T3,' AVG. NO. SH'
     6,'IP   SPEED '
     7,'RELATIVE SPEED   ALFA    PROB. DETECTION'/T5,'LEGS    '
     8,' SPEED RATIO UPLEG   DOWNLEG'//)
  601 FORMAT(T4,F5.2,2X,3(1X,F5.1),2X,F5.1,3X,F6.4,7X,F6.4)
  105 STOP
      END




      SUBROUTINE WYEB(XP,YP,PL,RO,BO,A)

C         INPUTS
C           YB=Y-POSITION OF SUB ON PATROL LINE
C           YB IS UNIFORMLY DISTRIBUTED
C           SGN=INITIAL DIRECTION (EQUALLY LIKELY) OF SUB
C           VELOCITY

      COMMON/ONE/SGN,YB
      DIMENSION A(2),B(2)

C         DETERMINE SUB POSIT RANDOMLY

      YB=YP+URN(1)*PL

C         DETERMINE INITIAL DIRECTION

    4 IF(URN(1)-0.5)5,6,7
    6 GO TO 4

C         UP

    5 SGN=1.
      GO TO 8

C         DOWN

    7 SGN=-1.

C         DETERMINE INITIAL RANGE AND BEARING TO SUB

    8 A(1)=XP
      A(2)=YB
      CALL VVAL(A,RO,B,BO,101)
```

```
      RETURN
      END




      SUBROUTINE CPACHK(R1,RN,UWV,RCPA,IN,J)
      DIMENSION R1(1),RN(1),UWV(1),X(2),Y(2),CPA(2),Z(2)
      COMMON/TWO/Y,N
C        R1=REFERENCE VECTOR
C        RN=TERMINAL VECTOR
C        UWV=UNIT VECTOR OF DRM

C        COMPUTE DOT PRODUCT: R1*UWV=X

      CALL VADD(R1,UWV,X,0)
      IF(X(1))1,2,2

C        CPA WILL OCCUR ON THIS LEG: COMPUTE THE DOT
C        PRODUCT: RN*UWV=X

    1 CALL VADD(RN,UWV,X,0)
      IF(X(1))3,3,4

C        CPA WILL NOT OCCUR ON THIS LEG: CPA=R1  COMPUTE
C        CPA

    2 CALL VVAL(R1,RCPA,X,D,100)
      RETURN

C        CPA WAS NOT REACHED ON THIS LEG:  CPA=RN: COMPUTE
C        CPA

    3 CALL VVAL(RN,RCPA,X,D,100)
      RETURN

C        DETERMINE CPA: ASSUME LEG HAS LENGTH< 250 MILES

    4 DO 5 I=1,500

C        USE HALF-MILE INCREMENTS

      P=I/2.

C        INCREMENT DRM BY 1/2 MILE

      CALL VMULT(UWV,Y,P)

C        ADD TO REFERENCE VECTOR

      CALL VADD(R1,Y,X,1)

C        COMPUTE DOT PRODUCT  X DOT UWV=Z

      CALL VADD(X,UWV,Z,0)

C        IF Z CHANGES SIGN, OR BECOMES ZERO VECTOR X=CPA:
C        OTHERWISE TRY AGAIN

      IF(Z(1)) 5,6,6
    5 CONTINUE

C        DRM LEG>250 MILES

      WRITE(6,60)IN,J
   60 FORMAT(//10X,'LEG',1X,I4,1X,'OF SAMPLE',I5,1X,'IS
```

```
     1GREATER THAN 250 MILES'//)
      RETURN
    6 CALL VVAL(X,RCPA,Y,D,100)
      RETURN
      END
```

```
      SUBROUTINE VMULT(A,B,X)
C
C        THIS SUBROUTINE MULTIPLIES THE VECTOR A BY THE
C        SCALAR X TO FORM THE VECTOR B
C
      DIMENSION A(1),B(1)
      DO 1 I=1,2
    1 B(I)=X*A(I)
      RETURN
      END
```

```
      SUBROUTINE VVAL(A,VL,UV,ANGLE,M)
C
C        THIS SUBROUTINE DETERMINES THE FOLLOWING
C        VALUES OF A:
C          A=INPUT VECTOR (SUPPLIED BY THE CALLING PROGRAM)
C          VL=MAGNITUDE OF A
C          UV=UNIT VECTOR PARALLEL TO A
C          ANGLE=ANGLE OF A MEASURED CCW FROM X-AXIS
C
      DIMENSION A(1),UV(1)
      IF(M-100)2,3,4
    4 IF(M-110)5,6,7
C
C        COMPUTE VECTOR LENGTH ONLY
C
    3 VL=SQRT(A(1)**2+A(2)**2)
      RETURN
C
C        COMPUTE ANGLE OF VECTOR ONLY
C
    2 ANGLE=ATAN2(A(2),A(1))
      RETURN
C
C        COMPUTE BOTH LENGTH AND ANGLE OF VECTOR
C
    5 VL=SQRT(A(1)**2+A(2)**2)
      GO TO 2
C
C        GENERATE UNIT VECTOR ONLY
C
    6 VL=SQRT(A(1)**2+A(2)**2)
      DO 1 I=1,2
    1 UV(I)=A(I)/VL
      RETURN
C
C        COMPUTE ANGLE AND GENERATE UNIT VECTOR
C
    7 ANGLE=ATAN2(A(2),A(1))
      GO TO 6
      END
```

```
      SUBROUTINE VINS(A,X,Y)
C        THIS SUBROUTINE INSERTS THE X AND Y VECTOR
C        COMPONENTS INTO ARRAY A
      DIMENSION A(1)
      A(1)=X
      A(2)=Y
      RETURN
      END
```

```
      SUBROUTINE COPI(A,B)
C        THIS SUBROUTINE GENERATES THE VECTOR A SUCH THAT
C        FOR A GIVEN VECTOR B, A=B
      DIMENSION A(1),B(1)
      DO 1 I=1,2
    1 A(I)=B(I)
      RETURN
      END
```

```
      SUBROUTINE VADD(A,B,R,N)
C        THIS SUBROUTINE DOES THE FOLLOWING VECTOR
C        OPERATIONS:
C           IF N>0: R=A+B
C           IF N<0: R=A-B
C           IF N=0: THE DOT PRODUCT OF A AND B IS FORMED AND
C        RESULT IS PLACED IN THE FIRST MEMBER OF ARRAY R
      DIMENSION A(1),B(1),R(1)
      IF(N)1,2,3
C        ADD VECTORS
    3 DO 4 I=1,2
    4 R(I)=A(I)+B(I)
      RETURN
C        SUBTRACT VECTORS
    1 DO 5 I=1,2
    5 R(I)=A(I)-B(I)
      RETURN
C        COMPUTE DOT PRODUCT OF VECTORS
    2 DO 6 I=1,2
    6 R(I)=A(I)*B(I)
      R(1)=R(1)+R(2)
      RETURN
      END
```

```
C          THIS SUBROUTINE GENERATES N(0,SIGMA SQUARED)
C          RANDOM VARIABLES BY THE BOX-MUELLER METHOD

C          TWO INDEPENDENT NORMALLY DISTRIBUTED RANDOM
C          VARIABLES ARE PRODUCED EACH TIME THIS SUBROUTINE
C          IS CALLED

C          D=STANDARD DEVIATION

      REAL FUNCTION NORM(D)
      DATA N,TWOPI/1,6.2831853/
      GO TO (1,2),N
    1 R=SQRT(-2.*ALOG(URN(1)))
      ARG=TWOPI*(URN(1))
      NORM=R*COS(ARG)*D
      X2=R*SIN(ARG)*D
      N=2
      RETURN
    2 NORM=X2
      N=1
      RETURN
      END
```

APPENDIX E

A. GEOMETRIC INTEGRATION

Figure 7 is a graph of a family of curves plotted from Table E-1a which represents the conditional probability that detection occurs as a function of the crossing point displacement from the patrol line center. The normal density function, with mean zero and standard deviation 20 miles, is also plotted in the figure. The upper set of numbers along the abscissa represents the normalized values, i.e.,

$$x' = x/20$$

where x' is the normalized value of x. The product of the normal density curve with each of the conditional probability curves is tabulated in Table E-1b, and plotted as a family of curves in Figure 9. If a grid of 0.1 millimeter squares is superimposed on this figure, the area represents one-half of the integral of the function. The area under each curve was determined by the grid-counting method, and was also determined by use of the Amsler Polar Planimeter. Each curve was traced five times with the planimeter stylus, and an average planimeter dial reading taken. The results of these two methods of geometric integration are tabulated in Table 9. The average planimeter dial readings were normalized to 0.998 (the value of $P(D)$ for $\rho = 0$) using the following equation:

TABLE E-1

CONDITIONAL PROBABILITY OF DETECTION
GIVEN CROSSING POINT IS DISPLACED
Y MILES FROM PATROL CENTER

a. NO ASSUMPTION ON DISTRIBUTION OF CROSSING POINTS

y	$P(D\|Y_p = y)$ SHIP-SUBMARINE SPEED RATIO (ρ)							
	$\rho=0$	$\rho=1$	$\rho=1.5$	$\rho=2$	$\rho=3$	$\rho=5$	$\rho=9$	$\rho=\infty$
0	1	1	.9354	.8000	.6250	.5104	.5021	.5000
5	1	1	.9260	.7844	.6250	.5260	.5021	.5000
10	1	1	.8687	.7635	.6177	.5469	.5021	.5000
15	1	.9781	.8062	.7125	.6104	.5604	.5292	.5000
20	1	.9365	.7437	.6500	.5969	.5604	.5292	.5000
25	1	.8844	.6812	.5875	.5552	.5469	.5292	.5000
30	1	.8219	.6187	.5302	.5135	.5052	.5010	.5000
50	1	.5719	.3844	.3635	.3469	.3385	.3344	.3333
70	1	.3219	.2177	.1969	.1802	.1719	.1677	.1667

b. NORMAL DISTRIBUTION (STANDARD DEVIATION 20 MILES) OF CROSSING POINTS ASSUMED

y	$\phi(y)$*	$P(D\|Y_p = y)$ SHIP SUBMARINE SPEED RATIO (ρ)						
	$\rho=0$	$\rho=1$	$\rho=1.5$	$\rho=2$	$\rho=3$	$\rho=5$	$\rho=9$	$\rho=\infty$
0	.3989	.3989	.3731	.3191	.2493	.2036	.2003	.1994
5	.3867	.3867	.3581	.3033	.2417	.2034	.1942	.1934
10	.3521	.3521	.3059	.2688	.2175	.1926	.1768	.1760
15	.3011	.2945	.2428	.2145	.1838	.1687	.1574	.1506
20	.2420	.2266	.1800	.1573	.1444	.1356	.1281	.1210
25	.1826	.1615	.1244	.1073	.1014	.0999	.0966	.0913
30	.1295	.1064	.0812	.0687	.0665	.0654	.0649	.0648
50	.0175	.0100	.0067	.0064	.0061	.0059	.0058	.0058
70	.0009	.0006	.0002	.0002	.0004	.0002	.0002	.0003

*Extracted from CRC Standard Mathematical Tables, 16th ed., 1968, pp. 549-555.

Area under Curve = (0.9980 x average reading)/762 (2)

where the constant 762 is the average planimeter dial reading obtained by tracing the curve for $\rho = 0$. The values contained in Table E-2 for the column under the heading "Planimeter P(D)" were computed by equation (2) using a Friden electronic calculator.

TABLE E-2

RESULTS OF TWO GEOMETRICAL
INTEGRATION METHODS

SPEED RATIO (ρ)	P (DETECTION)	
	GRID COUNTING	AMSLER POLAR PLANIMETER *
0	0.9980	0.9980
1	0.9415	0.9319
1.5	0.7895	0.7922
2	0.6816	0.6828
3	0.5781	0.5797
5	0.5233	0.5234
9	0.5008	0.5011
∞	0.4885	0.4883

*The planimeter dial readings were normalized so that 762 increments on the dial represented the known area 0.9980.

APPENDIX F

LIMITING CASE FOR P(DETECTION)

```
C
C
C          LIMIT
C
C
      J=0
      R=30.
      PL=120.
      YP=40.
      WRITE(6,600)R,PL
      Z=YP+PL
      DO 1 I=1,41
      IF(YP-R)3,2,2
    2 P=0
      GO TO 4
    3 IF(YP+R)5,5,6
    5 P=(2.*R)/PL
      GO TO 4
    6 Y=AMIN1(ABS(YP),ABS(Z))
      S=(YP*Z)
      W=S/ABS(S)
      P=(R-W*Y)/PL
    4 J=J+1
      IF(J-6)7,8,9
    8 J=0
      WRITE(6,602)
    7 WRITE(6,601)YP,P
    1 YP=YP-2.5
    9 STOP
  600 FORMAT(1H1,15(/),20X,'LIMITING CASE FOR P(DETECTION)'/
     1//5X,'DETECTION RADIUS (DEFINITE RANGE LAW): ',F4.0//5
     2X, 'PATROL LENGTH: ',F4.0///7X,'YP',8X,'P(D)'//)
  601 FORMAT(5X,F5.1,5X,F6.4)
  602 FORMAT (/)
      END
```

TABLE F-1

LIMITING CASE FOR P(DETECTION)

DETECTION RADIUS (DEFINITE RANGE LAW): 30.

PATROL LENGTH: 120.

YP	P(D)
40.0	0.0
37.5	0.0
35.0	0.0
32.5	0.0
30.0	0.0
27.5	0.0208
25.0	0.0417
22.5	0.0625
20.0	0.0833
17.5	0.1042
15.0	0.1250
12.5	0.1458
10.0	0.1667
7.5	0.1875
5.0	0.2083
2.5	0.2292
0.0	0.2500
-2.5	0.2708
-5.0	0.2917
-7.5	0.3125
-10.0	0.3333
-12.5	0.3542
-15.0	0.3750
-17.5	0.3958
-20.0	0.4167
-22.5	0.4375
-25.0	0.4583
-27.5	0.4792
-30.0	0.5000
-32.5	0.5000
-35.0	0.5000
-37.5	0.5000
-40.0	0.5000
-42.5	0.5000
-45.0	0.5000
-47.5	0.5000
-50.0	0.5000
-52.5	0.5000
-55.0	0.5000
-57.5	0.5000
-60.0	0.5000

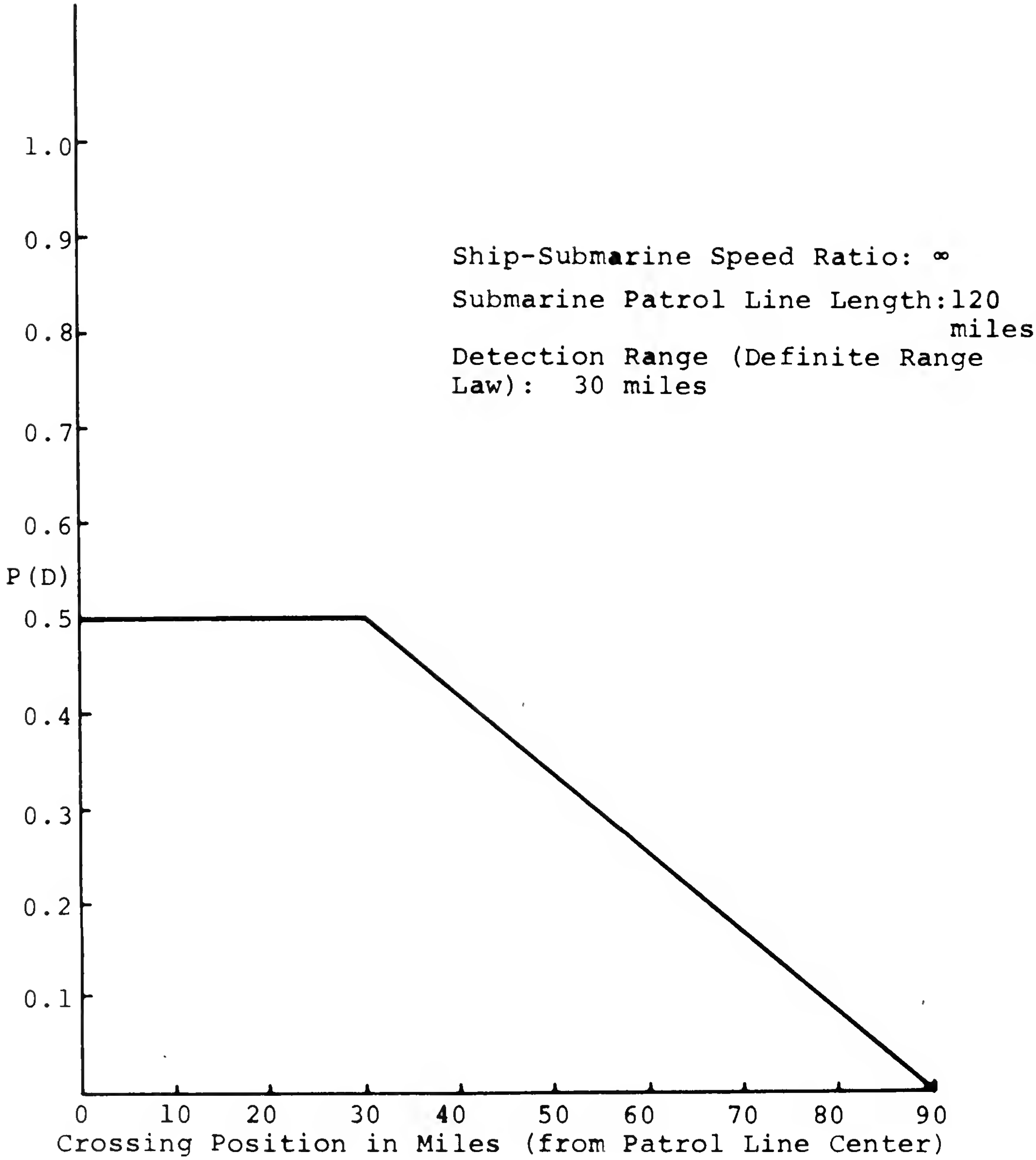


Figure F-1. Probability of Detection vs. Crossing Position (Perpendicular Tracks) for the Limiting Case

FOOTNOTES

1. Operations Evaluation Group, Office of the Chief of Naval Operations, Study 548, The Effects of Increased Merchant Ship Speed on Vulnerability to Submarine Attacks (U), p. 31, February 28, 1955. (C)

2. Operations Evaluation Group, Office of the Chief of Naval Operations, Summary Report 10, Countering the Threat of the Nuclear-Powered Submarine (U), p. 3, April 30, 1962. (S)

3. Center for Naval Analyses, Operations Research Group, Report ORG(M) 44, An Analysis of U.S. Coastal Trade Convoys (U), p. 7, December 18, 1943. (C)

4. Operations Evaluation Group, Office of the Chief of Naval Operations, Report No. 56, Search and Screening, by B. O. Koopman, p. 106-107, 1946.

5. Ibid, p. 107, Figure 8.

6. Beckenbach, E. F., Modern Mathematics for the Engineer, p. 279, McGraw-Hill, 1956.

7. Operations Evaluation Group, Office of the Chief of Naval Operations, Study 647, The Vulnerability of Independent Merchant Ships (U), by Neufer, J. E., Appendix A, June 7, 1961. (S)

8. Koopman, op.cit. p. 107.

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13. ABSTRACT

Numerical integration and Monte Carlo techniques are used in the development of several models in order to determine the effect of probability of random detection of a merchant ship using speeds up to 90 knots by a 10-knot submarine patrolling a back-and-forth barrier. A definite range law for detection is assumed. Individual encounter models are developed for ship tracks which are extended to include the assumption of a normal distribution of crossing points. Computer programs of the models, written in the FORTRAN IV language, are included. The results are applied in a numerical example.

It is concluded that while increases in ship speeds do result in substantial decrease in probability of detection by a submarine in the case of a single barrier transit, a speed advantage alone when applied to a typical transit of the North Atlantic will not appreciably decrease the overall detection probability.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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BACK-AND-FORTH BARRIER						
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RANDOM ENCOUNTER						
INDEPENDENT MERCHANT SHIP						